(b) (i) Show that the momentum of a system in centre of mass frame is always zero.

(ii) Show that the relationship between the angular momentum relative to the centre of mass frame of reference of a system of particles and the angular momentum relative to the laboratory frame is

$$\vec{L} = \vec{L}_{CM} + \vec{r}_{CM} \times \vec{P}$$ 4+6

(c) (i) Assuming the earth as spherical, find the expression of its moment of inertia about its axis of symmetry.

(ii) If n and $(n+1)$ be the number of oscillations made by the standard and Kater's reversible pendulum respectively between two consecutive coincidences, then their respective time periods T_0 and T are related by the expression

$$T_0 n = T(n+1)$$

Show that if n is sufficiently large for a second pendulum (that is, $T_0 = 2$ seconds)

$$T = 2\left(1 - \frac{1}{n}\right)$$ 7+3

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2012

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. $\vec{A}$, $\vec{B}$, $\vec{C}$, ... etc., give an algebra. What does it mean? 1

2. (a) How can an elemental area be made a vector quantity? Give an idea that a vector quantity can be associated with electric current. 2

(b) How can a vector field be obtained from a scalar field $\phi(r)$? Can the frictional force be obtained from some potential that way? Give reasons. 1

3. *(a)* A function $\phi(r)$ that is even (or odd) under one of these space reflection operations ($x \leftrightarrow -x$ etc.), will remain even (or odd) after the ∇^2 operation but not after the $\vec{\nabla}$ operation. Explain. 2

(b) $\phi(r)$ is a scalar field. State whether the end result in the following cases is scalar or vector : 1

(i) $\nabla^2 \phi(r)$

(ii) $\nabla^2 [\vec{\nabla} \phi(r)]$

(c) Show that

$$\hat{i} \times (\vec{\nabla} \times \vec{r}) \neq (\hat{i} \times \vec{\nabla}) \times \vec{r}$$ 3

Or

If $\phi(r)$ and $\psi(r)$ are two scalar fields such that $\vec{\nabla} \phi(r) \times \vec{\nabla} \psi(r) = 0$ over all spaces, how are their equipotential surfaces and lines of force related?

4. *(a)* Give an idea of space curves. How is it useful in the study of kinematics? 2

(b) Show that

$$\nabla^2 f(r) = \frac{d^2}{dr^2} f(r) + \frac{2}{r} \frac{d}{dr} f(r)$$ 5

(c) If $\vec{A}(r)$ is irrotational, show that $\vec{A}(r) \times \vec{r}$ is solenoid. 3

OR

5. *(a)* $\frac{d\phi}{ds} = \vec{\nabla}\phi \cdot \frac{d\vec{r}}{ds}$, where symbols are used in conventional meaning. Explain the terms present in the right-hand side of the expression. 2

(b) Let R be the distance from a fixed point $\vec{A}(a, b, c)$ to any point $P(x, y, z)$. Show that $\vec{\nabla} R$ is a unit vector in the direction $\overrightarrow{AP} = \vec{R}$. 3

(c) If $\vec{B}(r)$ is both irrotational and solenoidal, show that for a constant vector $\vec{m}$

$$\vec{\nabla} \times (\vec{B} \times \vec{m}) = \vec{\nabla}(\vec{B} \cdot \vec{m})$$ 5

GROUP—B

(Mechanics)

(*Marks* : 40)

6. (*a*) Can a frame of reference be the source of force? Explain. 1

(*b*) Observing a vector $\vec{A}$ from a rotating frame of reference, write its total time derivative. 1

(*c*) State the property of time on which the conservation of mechanical energy rests. 1

(*d*) Is the centre of mass frame of reference an inertial frame? Explain. 1

(*e*) What is principal moment of inertia of a rigid body? 1

(*f*) Due to Tsunami, the duration of the day and night of the earth is changed. Give a simplest explanation of this effect in terms of moment of inertia. 1

7. (*a*) Give schematic diagrams of the two particles collision in laboratory frame and centre of mass frame. 2

(*b*) The force $\vec{F} = (2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}$ is a conservative force. Find its potential function. 2

8. Answer any *two* questions : 5×2=10

(*a*) Find the kinetic energy of a system in its centre of mass frame.

(*b*) How can you compute the mass of a planet that has a satellite involving the time period of the satellite?

(*c*) Two particles having masses m_1 and m_2 travel along the x-axis with speeds u_1 and u_2 respectively. After collision their speeds become v_1 and v_2. Prove that the velocities of the centre of mass before and after collision remain same.

9. Answer any *two* questions : 10×2=20

(*a*) Establish the mathematical expression of acceleration of a particle observed in inertial frame relating the same acceleration observed in rotating frame of reference.

A satellite is moving in a circular polar orbit of radius R with uniform angular velocity ω. As the satellite moves towards the equator, it is observed by radar station situated at latitude λ north of equator. If earth rotates west to east at angular velocity Ω, find the velocity-expression of the satellite obtained by the radar station. 7+3 *(ii)* An achromatic converging combination of focal length 50 cm is formed with a convex lens of crown glass and a concave lens of flint glass placed in contact with each other. Calculate their focal lengths if the dispersive powers of the crown glass and flint glass are respectively 0·03 and 0·05. 3

(b) *(i)* With respective ray diagrams, explain what you understand by coma, astigmatism and distortion in case of aberration in optics. 6

(ii) Obtain the expression for lateral magnification of image produced by a convex lens. 4

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2012

PHYSICS

(Major)

Paper : 1.2

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 40)

1. *(a)* Indicate the type of motion described by the equation

$$m\ddot{x} = F_0 \cos pt - kx - R\dot{x}$$ 1

(b) What is the displacement of the particle in simple harmonic motion in one-time period? 1

(c) A sine wave is travelling in a medium. A particular particle has zero displacement at a certain instant. What is the displacement of the particle closest to it having zero displacement? 1

(d) When a sound wave is refracted from air to water then which of the following will remain unchanged? 1

Wave number, wavelength, wave velocity, frequency

(e) On the basis of absorption coefficient, distinguish between live room and dead room. 1

(f) When a particle is subjected to two simple harmonic motions $x = a\cos\omega t$ and $y = b\cos(\omega t + \alpha)$ at right angles to each other, it follows a uniform circular motion. What is the value of α and relation between a and b? 1

2. *(a)* A simple harmonic oscillator of mass 0·2 g has an amplitude 4 cm. Its velocity at zero displacement is 1 ms^{-1}. Find the frequency and the energy of oscillation. 2

(b) Distinguish between phase velocity and group velocity. 2

3. Answer any *two* questions : 5×2=10

(a) A microphone emits a 1 kHz pure tone having an intensity level of 65 dB. Calculate the actual intensity (reference intensity = 10^{-12} Wm^{-2}) and the loudness level of the sound. When another source of the same intensity is switched on, calculate the increase in the intensity level in dB.

(b) Determine the velocity of longitudinal waves in a thin solid rod in terms of Young's modulus and the density of the material of the rod.

(c) Deduce the expression for the energy of the string vibrating transversely.

4. *(a)* Write the differential equation for the damped simple harmonic oscillation of a system from the energy principle and solve it. Discuss the condition for underdamped motion. Draw the time-displacement graph for this case.

2+3+3+2=10

Or

What is a stationary wave? Discuss how a stationary wave is formed due to the superposition of two plane harmonic waves of same amplitude and frequency, propagating in opposite directions. Show that in a stationary wave the pressure nodes coincide with the displacement antinodes and vice versa. 1+5+4=10 *(b)* State Fourier's theorem. What simplification is obtained in the Fourier series if the function is odd? Analyse, with the help of Fourier series, a waveform given by

$$f(t) = \frac{A}{T}t, \quad \text{for } 0 < t < T$$

(A = constant)

Also plot the Fourier synthesis with first four terms. 2+1+5+2=10

Or

Define eigenfunctions, eigenvalues and eigenfrequencies for transverse vibration of a stretched string. Obtain the amplitudes of the different frequencies of a uniformly stretched string with two ends fixed when it is struck over a small region at a distance h from one end. Assume that the region moves with the instantaneous velocity v at time $t = 0$. 3+7=10

SECTION—II

(*Marks* : 20)

5. What is the meaning of an achromatic system? 1

6. *(a)* In the matrix formalism what advantage do we get if we consider the lens to be thin? 2

(b) What do you mean by aplanatic surface? 2

7. Answer any *one* question : 5

(a) Using matrix method, find the equivalent focal length of two lenses in contact in air of focal lengths f_1 and f_2.

(b) Establish Fermat's principle from refraction of light at a spherical surface.

8. Answer any *one* question :

(a) *(i)* Explain the causes of chromatic aberration of lens. Deduce an expression for longitudinal chromatic aberration. 2+5=7