2014

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. *(a)* How do you define the direction of angular momentum? Why is the Higgs particle termed as Higgs scalar not Higgs vector? 1+1=2

(b) Give the vector diagram representation of $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{B} \times \vec{A} = \vec{D}$. Name a vector physical quantity which is the product of two vectors. 1+1=2

(c) The electric field intensity $\vec{X}$ at any point is in the direction of the maximum rate of decrease of potential ϕ. Express it in the mathematical form. Give the mathematical expression of the divergence of a vector over the surface of a unit volume element surrounding a point in a vector field. 1+1=2

(d) Is the direction of a vector an absolute concept? Explain using the idea of derivative of a vector. 1+1=2

(e) The Euclidean space cannot be considered as a vector space. Why? Show how you get a scalar field from a vector field. 1+1=2

2. (a) Show that the gradient of any scalar field $\phi(r)$ is irrotational and the curl of any vector field $\vec{V}(r)$ is solenoidal. 4

(b) Prove that

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

If $\vec{A}$ and $\vec{B}$ are irrotational, show that $\vec{A} \times \vec{B}$ is solenoidal. 6

Or

(c) (i) Let $\vec{r}(t)$ be a vector of fixed magnitude. Show that $\frac{d\vec{r}(t)}{dt}$ is perpendicular to $\vec{r}(t)$.

(ii) If $\vec{v}(t)$ is a time-dependent vector, show that

$$\vec{v} \cdot \frac{d\vec{v}}{dt} = v \frac{dv}{dt}$$

where v is the magnitude. How is this relation relevant in connection with the motion of a particle in a circular orbit? Explain. 2+(3+1)=6

GROUP—B

(Mechanics)

(*Marks* : 40)

3. (a) What is integral of the equation of motion? 1

(b) Can you state the relevance of rotating frame of reference with 'Mars Orbital Mission' of India? 1

(c) Give the hint that Tsunami can change the time duration of day and night. 1

(d) Which is the weakest interaction of all interactions known in nature? 1

(e) Centre of mass frame is also an inertial frame but all inertial frames cannot be considered as centre of mass frame to investigate a particular physical system. Why? 1

(f) What is the meaning of the statement that there is no absolute velocity? Explain whether the momentum is a frame-dependent quantity or not. 1

4. (a) Show that the moment of inertia of the body about an axis is numerically equal to the torque producing unit angular acceleration about the same axis. 2

(b) Show that the gravitational force is conservative. 2

5. Answer any *two* questions : 5×2=10

(a) Calculate the centre of mass of semicircular arc and semicircular disc.

(b) Derive an expression for the gravitational field inside a sphere of radius R when the mass density at a point is $\rho = a + br^2$ where r is the distance of the point from the centre of the sphere, a and b are two constants.

(c) Find out the mathematical expression of moment of inertia of a spherical shell about a diameter as axis.

6. Answer any *two* questions : 10×2=20

(a) What is Galilean transformation equation? Is it relevant to non-inertial frame of reference? Explain. Derive the equation of motion in a rotating frame of reference. 1+1+8=10

(b) Give the theory of the compound pendulum and show that the centres of suspension and oscillation are reversible. Derive the condition for minimum time period. 3+4+3=10

(c) Establish the relations between scattering angles in Lab and CM frames. Considering the elastic collision, show that

$$\tan \theta_1 = \frac{\sin \theta_c}{\cos \theta_c + \frac{m_1}{m_2}}$$

where the symbols carry usual meanings. 8+2=10

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2014

PHYSICS

(Major)

Paper : 1.2

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 40)

1. *(a)* If two waves are represented by $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x' = a\cos\omega t$, then what is the phase difference between the two waves? 1

(b) What is the arithmetic intensity ratio of sound wave corresponding to a change of 1 dB? 1

(c) What is the ratio between the intensities of first and third harmonics produced in a spring plucked at the midpoint? 1

(d) Define a stationary wave. 1

(e) What simplification is obtained in the Fourier series if the function is even? 1

(f) What do you mean by the term 'optimum reverberation time'? 1

2. (a) "Initial conditions of a plucked string are static, while those of a struck string are dynamic." Justify the statement. 2

(b) Distinguish between phase velocity and group velocity. 2

3. Answer any *two* questions : 5×2=10

(a) Show that the acoustic intensity for a plane wave is the product of the r.m.s. sound pressure and the r.m.s. particle velocity.

(b) A massless spring whose upper end is fixed to a rigid support carries a horizontal disc of mass 100 g at the lower end. It is observed that the system oscillates with a frequency of 10 kHz and the amplitude of the damped oscillations reduces to half its undamped value in one minute. Calculate *(i)* the resistive force constant and *(ii)* its quality factor.

(c) Deduce the expression for the energy of a string vibrating transversely.

4. (a) Two simple harmonic motions act simultaneously on a particle at right angles to each other. Show that the path of the particle will be an ellipse when the two motions have the same period but different amplitudes and initial phases. What happens when the phase difference between the motions is *(i)* zero and *(ii)* $\frac{\pi}{2}$? 6+2+2=10

Or

(b) Explain mathematically how a stationary wave is formed due to superposition of two waves. Find the positions of displacement nodes and antinodes. Derive the expression of pressure to show that the pressure nodes coincide with the displacement antinodes. 4+3+3=10

(c) Analyze, with the help of Fourier theorem, a square periodic wave given by

$$f(t) = A \text{ (constant) for } 0 \le t \le \frac{T}{2}$$

$$= 0 \qquad \text{for } \frac{T}{2} \le t \le T$$

Also plot the Fourier synthesis with first four terms. 6+4=10

Or

(d) Find the expression of velocity of a longitudinal wave in a thin solid bar. Modify the expression using bulk modulus, rigidity modulus and Poisson's ratio so that it can be applicable for an extended solid medium. 6+4=10

SECTION—II

(*Marks* : 20)

5. State Fermat's principle of least action. 1

6. *(a)* What is achromatic doublet? 2

(b) In the matrix formalism, what advantage do we get if we consider the lens to be thin? 2

7. Find the condition of achromatism of two thin lenses separated by a small distance. 5

Or

Using matrix method, find the equivalent focal length of two lenses in contact in air of focal lengths f_1 and f_2.

8. Answer any *one* question :

(a) *(i)* Show that spherical refracting surface is aplanatic with respect to certain position of the object. 6

(ii) Distinguish between pin cushion and barrel-shaped distortion. 4

(b) *(i)* Establish Fermat's principle from refraction of light at a spherical surface. 5

(ii) Obtain the expression for lateral magnification of image produced by a convex lens. 5

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