where $C_{p0}, C_{p1}, \cdots, C_{pp}$ are unknown constants. Using orthogonality conditions, show that C_{pj} can be written as

$$C_{pj} = \frac{\begin{vmatrix} \mu_0 & \mu_1 \cdots -\mu_p \cdots & \mu_{p-1} \\ \mu_1 & \mu_2 \cdots -\mu_{p+1} \cdots & \mu_p \\ \vdots & & \\ \mu_{p-1} & \mu_p \cdots -\mu_{2p-1} \cdots & \mu_{2p-2} \end{vmatrix}}{\begin{vmatrix} \mu_0 & \mu_1 \cdots \mu_j \cdots & \mu_{p-1} \\ \mu_1 & \mu_2 \cdots \mu_{j+1} \cdots & \mu_p \\ \vdots & & \\ \mu_{p-1} & \mu_p \cdots \mu_{j+p-1} \cdots & \mu_{2p-1} \end{vmatrix}}$$ 10

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2014

STATISTICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions (reasoning is not necessary) : 1×7=7

(a) State the definition of mean deviation in words.

(b) State True or False for the following relationship :

Median = 5th decile ≠ 60th percentile

(c) State which of the following relationships are correct :

If $x_i / f_i (i = 1, 2, \cdots, n)$ is a frequency distribution, then—

(i) $\sum_{i=1}^{n} f_i (x_i - \bar{x}) = 0$

(ii) $\sum_{i=1}^{n} f_i |x_i - \bar{x}| = 0$

(d) State which of the statements is correct :

In drawing box-plot, we use

(i) range, median and standard deviation

(ii) first, second and third quartiles

(e) Mention any two measures of skewness.

(f) Give the definition of harmonic mean.

(g) State the advantage of coefficient of variation over standard deviation.

2. Answer the following questions : 2×4=8

(a) Write a brief note on standard deviation. 2

(b) Define first and third quartiles. 2

(c) Write a note on non-frequency data. 2

(d) Define rth raw and central moments and state the relationship between them. 1+1=2

3. Answer any *three* of the following : 5×3=15

(a) Prove that for any discrete distribution, standard deviation is least root-mean square deviation. 5

(b) Suppose you want to fit the equations of the types (i) $y = ab^x$ and (ii) $y = ax^b$. Explain how you would fit them by using the method of least square. 2½+2½=5

(c) Define mode and derive its formula. 1+4=5

(d) Prove that the correlation coefficient lies between zero and one. 5

(e) Write an explanatory note on Sheppard's correction for moments. 5

EITHER

4. (a) Compare and contrast between different measures of dispersion. 5

(b) Show that mean deviation is least when measured about median. 5

OR

5. (a) For the two variables X and Y, derive the line of regression of X on Y. 5

(b) Find the standard deviation of the AP series $a, a+d, a+2d, \cdots, a+2nd$. 5

EITHER

6. (a) (i) Define cumulants. State its properties.

(ii) Find the cumulant generating function of the random variable whose cumulants are

$$k_r = (r+1)!2^r$$ 1+2=3

(b) Show that the variance of weighted mean

$$\frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

of n independent random variables x_i is minimum when weights are inversely proportional to the corresponding variance. 7

OR

7\. *(a)* Show that if X', Y' are the deviations of the random variables X and Y from their respective means, then

$$r = 1 - \frac{1}{2N}\sum_i \left(\frac{X_i'}{\sigma_x} - \frac{Y_i'}{\sigma_y} \right)^2$$

N is the number of observations. 4

(b) If X and Y are uncorrelated random variables with means zero and variances σ_1^2 and σ_2^2 respectively, then find σ_U^2, σ_V^2, cov(U, V) and correlation coefficient between U and V where

$$U = X\cos\alpha + Y\sin\alpha$$
$$V = X\sin\alpha - Y\cos\alpha$$ 1+1+2+2=6

EITHER

8\. *(a)* In a frequency distribution, the coefficient of skewness based on the quartiles is 0·6. If the sum of the upper and lower quartiles is 100 and median is 38, find the value of quartile deviation. 4

(b) *(i)* If R is the range and σ is the standard deviation of a set of observations $x_1, x_2, \cdots, x_n$, then prove that $\sigma \le R$. 4

(ii) Write a note on factorial moments. 2

OR

9\. Explain the concept of orthogonal polynomials. Let P_p the polynomial of degree p in x be given by

$$P_p = \sum_{j=0}^{p} C_{pj} x^j$$

2014

STATISTICS

(Major)

Paper : 1.2

(Probability—I)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option/Write True or False of the following : 1×7=7

(a) Let A and B be two events. If

$$P(A \cup B) = P(A) + P(B)$$

then the events A and B are said to be

(i) independent

(ii) mutually exclusive

(iii) mutually independent

(iv) None of the above

(b) Let A be an event. Then probability of the event A, i.e., $P(A) \geq 0$.

(c) If A and B are two mutually exclusive events, then $P(A / A \cup B)$ is equal to

(i) $P(A)$

(ii) $\frac{P(A)}{P(A)+P(B)}$

(iii) $\frac{P(A \cup B)}{P(A)}$

(iv) None of the above

(d) Let X be a random variable. Then the distribution function of X, i.e., $F(x)$, always satisfies the relation $F(x) \leq 1$.

(e) Let X be a random variable having probability density function $f(x)$. Then the geometric mean of the random variable is represented by the relation

$$G = \int_{-\infty}^{\infty} \log x \, f(x) \, dx$$

(f) Let X be a random variable. Then the first factorial moment about origin and the first moment about origin are same.

(g) Let X be a random variable and a be any arbitrary value. Then $M_{X-a}(t)$ is equal to

(i) $M_X(at)$

(ii) $M_{aX}(t)$

(iii) $e^{-at} M_X(t)$

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) If X is a non-negative integer valued variate, then prove that

$$\sum_{k=1}^{\infty} kP(X > k) = \frac{1}{2}[E(X^2) - E(X)]$$

(b) A fair die is rolled twice. Let r be the event that the first shows a number ≤ 2 and B the event that the second throw shows at least 4. Describe the event $A \cap B$ and find $P(A \cup B)$.

(c) The distribution function F of a continuous random variable X is given by

$$\begin{aligned} F(x) &= 0 \quad , \quad x < 0 \\ &= x^2 \quad , \quad 0 \leq x \leq \tfrac{1}{2} \\ &= 1 - \frac{3(3-x)^2}{25} \quad , \quad \tfrac{1}{2} \leq x < 3 \\ &= 1 \quad , \quad x \geq 3 \end{aligned}$$

Find the p.d.f. of X with comments.

(d) From an urn containing a white and b black balls, a certain number of k balls is drawn and not replaced back, then a ball is drawn from the urn. What is the probability that this is white ball?

3. Answer any *three* of the following questions : 5×3=15

(a) A bowl contains four balls, identically in all respects, numbered 1, 2, 3, 4. A ball is chosen at random. Events are defined A_1, A_2 and A_3 as follows :

Event A_i occurs iff the chosen ball is numbered either i or 4; $i = 1, 2, 3$

(i) Examine the independence of A_1, A_2 and A_3. 3

(ii) Describe in words the events

$$(A_1 \cup A_2) \cap A_3$$ 2

(b) An urn contains N balls among which W are white. A random sample of n is drawn without replacement and from this sample another random sample of size m is drawn without replacement. Find that the second sample contains exactly k white balls. 5

(c) A communication system consists of n components each of which will independently function with probability p. The total system will be able to operate effectively if at least one-half of the components function. For what value of p is a 5-component system more likely to operate effectively than a 3-component system? 5

(d) For a random variable X, prove that

$$E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$$ 5

(e) Concentric circles of radius 1 cm and 3 cm are drawn on a circle of radius 5 cm. A man receives 10, 5 or 3 points if he hits the target inside the smaller circle, inside the middle angular region or inside the outer annular region respectively. Suppose the man hits the target with probability $\frac{1}{2}$ and then is just as likely to hit one point of the target as the other. Find the expected number E of points he scores each time he fires. 5

4. Answer any *three* of the following questions : 10×3=30

(a) A random variable X has distribution function

$$F(x) = \begin{cases} 0 & , \ x < 0 \\ \frac{1-\cos x}{2} & , \ 0 \leq x < \pi \\ 1 & , \ x > \pi \end{cases}$$

(i) Find the expectation of X.

(ii) Find the variance of X.

(iii) Find the median of X.

(iv) Find the mode of X. 2½×4=10

(b) Let X be a random variable with probability density function

$$f(x) = c(1-x), \ 0 < x < 1$$

Find *(i)* the value of c, *(ii)* μ_2, μ_3 and μ_4, and *(iii)* β_1 and β_2. 10

(c) An urn contains N balls of which M are white. A sample of n balls are drawn from the urn. Let A_k be the event that the sample contains exactly k white balls and B_j be the event that the jth ball is white. Find $P(B_j / A_k)$ when the sample is drawn *(i)* without replacement and *(ii)* with replacement. 10

(d) Let the probability P_N that a family has n children be αp^n, $n \geq 1$ and

$$p_0 = 1 - \alpha p(1 + p + p^2 + \cdots)$$

(i) Show that for $k \geq 1$ the probability that a family contains exactly k boys is $2\alpha p^k / (2-p)^{k+1}$.

(ii) Given that a family includes at least one boy, show that the probability that there are two or more boys is $p/(2-p)$. 10

(e) Define characteristic function and describe its properties. Obtain cumulant generating functions and its properties, and hence obtain cumulants. 10

(f) *(i)* Define the probability—classical, relative frequency approach and axiomatic approach. Discuss their advantages and disadvantages with examples.

(ii) State and prove compound probability rules. 10

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