2013

MATHEMATICS

(Major)

Paper : 2.1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* What will be the equation of the line $x+y=2$, when the origin is transferred to the point (1, 1)? 1

(b) Write down the condition for pair of lines represented by the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$ 1

(c) Write down the parametric equations of the parabola, $y^2=4ax$. 1

(d) What are the direction ratios of the normal to the plane, $x+y+z=0$? 1

(e) Write down the direction cosines of z-axis. 1

(f) What conic does the following equation represent? 1

$$x^2 + 2xy + y^2 - 2x - 1 = 0$$

(g) Write down the equation of the tangent to

$$\frac{l}{r} = 1 + e\cos\theta$$

at α. 1

(h) What are the centre and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$$ 1

(i) Define skew lines. 1

(j) Under what condition, the lines represented by $ax^2 + 2hxy + by^2 = 0$ will be perpendicular to each other. 1

2. (a) Transform the equation $x^2 - y^2 = a^2$ by taking the perpendicular lines

$$y - x = 0 \text{ and } y + x = 0$$

as coordinate axes. 2

(b) Find the equation of the sphere through circles $x^2 + y^2 + z^2 = 25$, $x + 2y - z + 2 = 0$ and the point (1, 1, 1). 2

(c) The axis of a right circular cylinder is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$

and its radius is 5. Find its equation. 2

(d) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola, $y^2 = 4ax$, prove that

$$t_1 t_2 = -1$$ 2

(e) Find the equation of the plane containing the lines

$$2x + 3y + 5z - 7 = 0,\ 3x - 4y + z + 14 = 0$$

and passing through the origin. 2

3. (a) A sphere of constant radius, r passes through the origin, O and cut the axes at A, B and C. Prove that the locus of the foot of the perpendicular from O to the plane ABC is

$$(x^2 + y^2 + z^2)^2 (x^{-2} + y^{-2} + z^{-2}) = 4r^2$$ 5

(b) Prove that the tangents at the extremities of a diameter of an ellipse are parallel to the conjugate diameter. 5

(c) Find the length of the semi-axes of the conic

$$ax^2 + 2hxy + ay^2 = d$$ 5

Or

Show that the locus of the middle points of the normal chords of the parabola, $y^2 = 4ax$ is

$$y^4 - 2a(x-2a)y^2 + 8a^4 = 0$$

(d) Prove that the lines $x = pz + q$, $y = rz + s$ intersect the conic

$$z = 0, \ ax^2 + by^2 = 1$$

if $aq^2 + bs^2 = 1$. 5

Or

Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines, if

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

4. (a) Find the equation of the asymptotes of a central conic given by the general equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

and show that the angle between the asymptotes is

$$\tan^{-1}\left[\frac{2\sqrt{h^2 - ab}}{a+b}\right]$$ 6+4=10

Or

Obtain the equation of the chord of the conic

$$\frac{l}{r} = 1 + e\cos\theta$$

joining the two points on the conic whose vectorial angles are

$$\alpha + \beta \text{ and } \alpha - \beta$$

Hence deduce the equation of the tangent at the point, whose vectorial angle is α. 8+2=10

(b) Obtain the length and equations of the shortest distance between the lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

$$\text{and } \frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$$

Find the condition for which the lines are coplanar. 6+3+1=10

(c) Find the condition that the plane

$$lx + my + nz = p$$

may be a tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$ and find the coordinates of the point of contact. Also find the equation of the tangent planes to the conicoid. 6+2+2=10

Or

Find the area of the section of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

by the plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$ 10

(d) Show that the equation

$$(ab-h^2)(ax^2+2hxy+by^2+2gx+2fy) + af^2+bg^2-2fgh=0$$

represents a pair of straight lines and that these straight lines form a rhombus with the line

$$ax^2+2hxy+by^2=0$$

provide that

$$(a-b)fg+h(f^2-g^2)=0$$ 10

Or

The origin of a rectangular coordinate system is translated to the new origin (α, β) and then the axes are rotated

through an angle θ. If (x, y) and (x', y') are the coordinates of a point with respect to the original and the transformed coordinate system, respectively then show that

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

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2013

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

(a) What do you mean by order and degree of a differential equation?

(b) Determine if the following differential equation is homogeneous :

$$y_1 = \frac{x^2 + y}{x^3}$$

(c) Write down the general form of a linear differential equation of order two.

(d) Write down the integrating factor of the differential equation :

$$(1+x^2)y_1 + 2xy - 4x^2 = 0$$

(e) Find the particular integral of the differential equation

$$(D^2 - 4)y = \sin 2x$$

(f) When is the total differential equation

$$Pdx + Qdy + Rdy = 0$$

(P, Q, R are functions of x, y and z) said to be exact?

(g) Write down the general form of a linear partial differential equation of order one with n independent variables.

(h) Write down the general solution of the differential equation

$$p^2 - 5p + 6 = 0;\ p = \frac{dy}{dx}$$

(i) What do you mean by trajectory of a given family of curves?

(j) What is complementary function of the differential equation

$$(D^2 + 4D + 4)y = x^3\ ?$$

2. Answer the following : 2×5=10

(a) Solve :

$$\frac{xdx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}$$

(b) Solve :

$$\frac{dy}{dx} = (4x + y + 1)^2$$

(c) Solve :

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$$

(d) Solve :

$$(2x^3 y + 1)dx + x^4 dy + x^2 \tan z dz = 0$$

(e) Construct the partial differential equation by eliminating a, b and c from $z = a(x+y) + b(x-y) + abt + c$.

3. Answer any *four* parts : 5×4=20

(a) Solve

$$(px - y)(py + x) = h^2 p$$

using the transformation $x^2 = u$ and $y^2 = v$.

(b) Show that the following equation is exact and hence solve it

$$(1+e^{x/y})dx + e^{x/y}(1-\frac{x}{y})dy = 0$$

(c) Solve :

$$x^2D^2y - 3xDy + 5y = x^2 \sin\log x$$

(d) Solve by the method of variation of parameter :

$$y_2 + 4y = 4\tan 2x$$

(e) Find the differential equation of all spheres of radius c having centre in the xy-plane.

(f) Solve by Lagrange's method :

$$(y^2+z^2-x^2)p - 2xyq + 2zx = 0$$

4. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Show that the system of confocal and co-axial parabolas $y^2 = 4a(x+a)$ is self-orthogonal, a being parameter.

(b) Solve :

$$\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$$

(c) Solve :

$$(D^3+8)y = x^4 + 2x + 1$$

(d) Show that $Ax^2 + By^2 = 1$ is the solution of

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

5. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve :

$$x^2y_2 - (x^2+2x)y_1 + (x+2)y = x^3e^x$$

(b) Transform the differential equation

$$\cos xy_2 + \sin xy_1 - 2y\cos^3 x = 2\cos^5 x$$

into one having z as independent variable, where $z = \sin x$ and solve it.

(c) Reduce the differential equation

$$y_2 - 4xy_1 + (4x^2-1)y = -3e^{x^2}\sin 2x$$

to its normal form and hence solve it.

(d) Show that if z satisfies

$$\frac{d^2z}{dx^2} + p\frac{dz}{dx} = 0$$

by changing the independent variable x to z, we shall transform

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

into

$$\frac{d^2y}{dz^2} + Q_1y = R_1$$

where P, Q, R are functions of x.

6. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve :

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$$

(b) Show that the necessary condition for integrability of single differential equation $Pdx + Qdy + Rdz = 0$ is

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

(c) Solve :

$$\frac{dx}{dt} - 7x + y = 0$$
$$\frac{dy}{dt} - 2x - 5y = 0$$

(d) Find $f(z)$ such that

$$\left(\frac{y^2+z^2-x^2}{2x}\right)dx - ydy + f(z)dz = 0$$

is integrable. Hence solve it.

7. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve by Charpit's method :

$$2xz - px^2 - 2qxy + pq = 0$$

(b) Find the integral surface of the linear partial differential equation

$$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$$

which contains the straight line $x + y = 0,\ z = 1$.

(c) Derive the partial differential equation by the elimination of arbitrary function ϕ from the equation

$$\phi(u, v) = 0$$

where u and v are functions of x, y and z.

(d) Find the complete integral of

$$pq = 1$$

Find also the singular integral if it exists.

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