2013

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods II)

(*Marks* : 35)

1. Answer the following questions : 1×4=4

(a) If $\vec{F}$ is a conservative force field, find the value of $\oint_C \vec{F} \cdot d\vec{l}$ along a closed curve *C*.

(b) Write the mathematical statement for the flux of a vector $\vec{A}$ over a surface *S*.

(c) Write the expression for a vector $\vec{A}$ having covariant components c_1, c_2, c_3 in orthogonal curvilinear coordinate system (u_1, u_2, u_3).

(d) If

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x)\,dx = f(0)$$

find the value of $\int_{-\infty}^{+\infty} \delta(x)\,dx$.

2. Answer the following questions : 2×3=6

(a) The equation of motion of a particle of mass m is

$$m\frac{d^2\vec{r}}{dt^2} = f(r)\,\hat{r}$$

where $\vec{r}$ is the position vector of the particle with respect to the origin and $\hat{r}$ is its unit vector. Then show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = \vec{C}$$

is a constant vector.

(b) Evaluate

$$\int_2^3 \vec{A} \cdot \frac{d\vec{A}}{dt}\,dt$$

if $\vec{A}(2) = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{A}(3) = 4\hat{i} - 2\hat{j} + 3\hat{k}$.

(c) If

$$\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx \quad (n > 0)$$

find $\Gamma(1)$.

3. Show that in orthogonal curvilinear coordinates

$$\vec{\nabla} \cdot (A_1\hat{e}_1) = \frac{1}{h_1h_2h_3}\frac{\partial}{\partial u_1}(A_1h_2h_3)$$

where the symbols carry usual meanings. 5

Or

Prove that $\oint d\vec{r} \times \vec{B} = \iint_S (\hat{n} \times \vec{\nabla}) \times \vec{B}\,dS$ where $\hat{n}$ is the unit vector normal to the elemental area dS.

4. Answer *either (a) or* [*(b)* and *(c)*] :

Either

(a) State and prove Gauss' divergence theorem. 2+8=10

Or

(b) Express $\nabla^2\psi$ in orthogonal curvilinear coordinate system. 5

(c) If u_1, u_2 and u_3 are generalized orthogonal coordinates, then show that

$$\left(\frac{\partial\vec{r}}{\partial u_1}, \frac{\partial\vec{r}}{\partial u_2}, \frac{\partial\vec{r}}{\partial u_3}\right) \text{ and } (\vec{\nabla}u_1, \vec{\nabla}u_2, \vec{\nabla}u_3)$$

are reciprocal system of vectors. 5

5. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

Either

(a) Verify Stokes' theorem for

$$\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$$

taking S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 7

(b) Show that

$$\delta(ax) = \frac{1}{a}\delta(x), \qquad a > 0$$ 3

Or

(c) Calculate the scale factors h_1, h_2, h_3 in spherical polar coordinates. 4

(d) Show that

$$\Gamma\left(\frac{1}{2}\right) = \pi^{\frac{1}{2}}$$

Hence calculate $\Gamma\left(\frac{5}{2}\right)$. 5+1=6

GROUP—B

(Properties of Matter)

(*Marks* : 25)

6. Answer the following questions : 1×3=3

(a) What is the physical interpretation of negative value of Poisson's ratio?

(b) Why is the raindrop spherical?

(c) When a liquid in a tube is stirred and left itself, the motion subsides after sometime. For what phenomenon does it happen?

7. A steel rod of length 50 cm, width 2 cm and thickness 1 cm is bent in the form of an arc having radius 2·0 m. If Young's modulus of the material of the rod is 2×10^{11} N/m^2, calculate the bending moment. 2

8. Answer any *two* of the following questions : 5×2=10

(a) Show that a hollow cylinder is stronger than a solid one of the same mass, length and material.

(b) Find an expression for the volume of liquid flow per unit time through a combination of two capillary tubes of lengths and radii (l_1, r_1) and (l_2, r_2) joined smoothly in series.

(c) Derive an expression for excess pressure inside a curved surface of a liquid.

9. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] of the following questions :

Either

(a) Derive the relation $Y = 2n(1+\sigma)$, where Y, n and σ are Young's modulus, rigidity modulus and Poisson's ratio of a material respectively. 7

(b) A U-tube is supported with its limbs vertical and is partly filled with water. If the internal diameters of the limbs are 1 cm and 1 mm, what will be the difference in heights at which the water stands in two limbs? [Take surface tension of water = 70 dynes/cm, $g = 980$ cm/s^2 and density of water $\rho = 1$ gm/cc.] 3

Or

(c) Show that a cantilever clamped at one end and loaded at other end executes a simple harmonic oscillation, when it is depressed slightly and then released. 7

(d) In an experiment with Poiseuille's apparatus, the following readings were recorded :

Volume of alcohol flowing per minute $= 10^{-5}$ m^3

Density of alcohol $= 8 \times 10^2$ kg/m^3

Length of tube = 0·5 m

Radius of tube = 0·05 cm

Height of alcohol head = 0·6 m

Determine the coefficient of viscosity of the alcohol. 3

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2013

PHYSICS

(Major)

Paper : 2.2

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option/Answer the following : 1×7=7

(a) Why does the pressure of a gas on the container wall increase, when it is heated?

(b) A jar A is filled with a gas characterised by (p, V, T). Another jar B is filled with a gas with parameters $(2p, V/4, 2T)$. The ratio of the number of molecules in jar A to those in jar B is

(i) 1 : 1

(ii) 1 : 2

(iii) 2 : 1

(iv) 4 : 1

(c) The value of the critical volume V_c according to van der Waals equation is

(i) $V_c = 2b$

(ii) $V_c = b$

(iii) $V_c = 2 \cdot 5b$

(iv) None of the above

(d) With usual meanings of the symbols, the Einstein's equation for Brownian motion is given by

(i) $\Delta^2 = \frac{RT}{N_A} \frac{1}{3\pi\eta r}$ J

(ii) $\Delta^2 = \frac{KT}{3\pi\eta r\, N_A}$ J

(iii) $\Delta^2 = \frac{KT}{3\pi\eta Rr}$ J

(iv) None of the above

(e) The value of γ for an ideal monatomic gas is

(i) $\frac{3}{2}$

(ii) $\frac{5}{2}$

(iii) $\frac{3}{5}$

(iv) $\frac{5}{3}$

(f) Elaborate the essential difference between the first law and the second law of thermodynamics.

(g) What do you mean by 'lagged' bar? Is there any radiation loss in such a bar?

2. Answer the following questions : 2×4=8

(a) The density of hydrogen at NTP is $8 \cdot 96 \times 10^{-5}$ g/c.c. Calculate the root-mean-square velocity for an oxygen molecule at NTP.

(b) Callendar's formula regarding platinum temperature t_P is given by

$$t - t_P = K\left\{\left(\frac{t}{100}\right)^2 - \left(\frac{t}{100}\right)\right\}$$

Find an expression for K if

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

(c) Establish Stefan's law $E = \sigma T^4$ from Planck's radiation formula.

(d) Explain entropy of a thermodynamic system.

3. Answer any *three* of the following questions : 5×3=15

(a) Derive the relation

$$\left.\frac{\delta s}{\delta V}\right|_T = \left.\frac{\delta p}{\delta T}\right|_V$$

(b) If $H = U + pV$ represents enthalpy of a system containing a gas, prove that

$$C_P - C_V = p\left(\frac{\delta V}{\delta T}\right)_p + \left(\frac{\delta U}{\delta V}\right)_T \left(\frac{\delta V}{\delta T}\right)_p$$

(c) A cylindrical tube of radii r_1 and r_2 has temperatures θ_1 and θ_2 at the inner and outer surfaces respectively. Show that the temperature will be $\frac{1}{2}(\theta_1 + \theta_2)$ at a distance $\sqrt{r_1 r_2}$ from the axis.

(d) Deduce Clausius-Clapeyron equation from Maxwell's second thermodynamic relation.

(e) What is triple point? Discuss the thermodynamics of triple point.

4. (a) Derive Maxwell's law of distribution of velocities of the molecules of a gas. Find the ratio of the average velocity to r.m.s. velocity of the molecules. 10

Or

Deduce Planck's theory of black-body radiation and show analytically how this formula is used in longer as well as shorter wavelength ranges.

(b) What are degrees of freedom? State the law of equipartition of energy. Establish that associated energy per degree of freedom is $\frac{1}{2}KT$.

If the thermal energy of a thermodynamical system is $U = \frac{1}{2}NfKT$, where f is the number of degrees of freedom, find the value of $\gamma = \frac{C_P}{C_V}$ in terms of f. 1+1+6+2=10

Or

(i) State the second law of thermodynamics in terms of entropy.

(ii) Obtain an expression for the efficiency of Carnot's engine using a perfect gas as working substance. 2+8=10

(c) Deduce Kirchhoff's law of radiation. 10

Or

Write short notes on (any *two*) : 5×2=10

(i) Rayleigh-Jeans law

(ii) Adiabatic demagnetization

(iii) Fourier equation for rectilinear flow of heat

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