2013

STATISTICS

(Major)

Paper : 2·1

(Numerical and Computational Techniques—I)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×7=7

(a) The third difference of zero of third degree is

(i) zero

(ii) 1

(iii) 6

(iv) None of the above

(b) The nth difference of constant is

(i) 1

(ii) zero

(iii) $n!$

(iv) None of the above

(c) "The divided difference is a symmetrical function of all the arguments involved and follows that for any function, $f(x)$ the value of a divided difference remains unaltered when any of the arguments involved are interchanged."

(i) The statement is True

(ii) The statement is False

(iii) The statement is neither True nor False

(iv) None of the above

(d) The correct relation is

(i) $\Delta \equiv 1 - E^{-1}$

(ii) $\nabla \equiv 1 - E^{-1}$

(iii) $E^{1/2} \equiv 1 - \nabla$

(iv) All of the above

Here ∇ and E denote the backward difference and shift operators, respectively.

(e) Simpson's one-third rule is used

(i) for numerical integration

(ii) to determine the roots of a polynomial

(iii) to solve both (i) and (ii)

(iv) None of the above

(f) Regula falsi method is used for

(i) the determination of the roots of a polynomial

(ii) solution for differential equation

(iii) numerical integration

(iv) None of the above

(g) A difference equation is an equation which involves

(i) independent variable

(ii) dependent variable

(iii) the successive differences of the dependent variable

(iv) All of the above

2. Answer the following : 2×4=8

(a) Prove for positive integer m,

$$\Delta^2 x^{(m)} = m(m-1)x^{(m-2)}$$

(b) Prove the operator relation,

$$(1+\Delta)(1-\nabla) = 1$$

(c) Solve :

$$u_{x+1} - u_x = (x^2 - 2x)2^x$$

(d) Find the real root of

$$2x - \log_{10} x = 7$$

Answer any *three* questions : 5×3=15

3. Show that

$$B(m+1, n) = (-1)^m \Delta^m \left(\frac{1}{n}\right)$$

where m is a positive integer.

4. Find by the method of iteration a real root of $2x - \log_{10} x = 7$.

5. Evaluate $\int_0^1 \frac{dx}{1+x}$ correct up to five decimal places by Euler-Maclaurin formula.

6. Solve :

$$u_{x+1}u_x + (x+2)u_{x+1} + xu_x = -2 - 2x - x^2$$

7. Assuming Bessel's interpolation formula, obtain the following result :

$$\frac{d}{dx}\frac{y}{x} = \Delta y_{x-1/2} - \frac{1}{24}\Delta^3 y_{x-3/2} + \cdots$$

Answer any *three* questions : 10×3=30

8. State and derive Newton-Gregory formula for backward interpolation.

9. State and derive three methods for Lagrange's interpolation formula for unequal intervals.

10. State and derive Laplace-Everett formula for interpolation.

11. Explain how Newton-Gregory formula can be used to carry out inverse interpolation of second order. Show that a good approximation to the interpolating factor, x will be given by

$$x = \frac{f_x - f_0}{\Delta f_0}\left[1 - \left\{\frac{f_x - f_0}{2(\Delta f_0)^2}\right\}\Delta^2 f_0\right]$$

12. Obtain approximations to the value of or evaluate

$$\int_0^6 \frac{dx}{1+x^2}$$

by applying *(a)* Weddle's rule, *(b)* Simpson's one-third rule and compare the results with the true value, $\tan^{-1} 6 = 1 \cdot 406$.

13. *(a)* Solve the equation $u_{x+2} - 4u_x = 9x^2$.

(b) Show that

$$\Delta O^m - \frac{1}{2}\Delta^2 O^m + \frac{1}{3}\Delta^3 O^m - \cdots O, \text{ if } m > 1.$$

★ ★ ★

(c) (i) Show that

$$\int_a^b (x-a)^{l-1}(b-x)^{m-1}\,dx = (b-a)^{l+m-1}\beta(l, m)$$ 5

(ii) Write a note on Lagrange's undetermined multipliers. 5

(d) (i) Show that if $2x+3y+4z=a$, the maximum value of

$$x^2y^3z^4 \text{ is } \left(\frac{a}{9}\right)^9$$ 5

(ii) Show that

$$\int_0^p x^m (p^q - x^q)^n\,dx = \frac{p^{qn+m+1}}{q}\beta\left(n+1, \frac{m+1}{q}\right)$$

if $p>0$, $q>0$, $m+1>0$, $n+1>0$. 5

(e) (i) Prove that

$$\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} = \beta(m, n)$$ 5

(ii) Prove that

$$\int_0^\infty \frac{(x^{m-1}+x^{n-1})}{(1+x)^{m+n}}\,dx = 2\beta(m, n)$$ 5

(f) (i) Prove that

$$\Gamma(n+1) = n! \quad \forall\ n \in N$$ 5

(ii) Show that, for $m, n > 0$

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}}\,dx = \frac{\beta(m, n)}{(b+c)^m b^n}$$ 5

★★★

2013

STATISTICS

(Major)

Paper : 2.2

(Mathematical Methods—I)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×7=7

(a) State the Taylor's expansion for one variable.

(b) Define an explicit function.

(c) In connection with the Reimann integral, define the term 'refinement'.

(d) Define the Gamma integral.

(e) State when a function, $f(x, y)$ is said to tend a limit, l.

(f) If $f(x, y) = 2x^4 - 3x^2y + y^2$, find $f_{xy}(x, y)$ at $(0, 0)$.

(g) Define the improper integral.

2. Answer the following : 2×4=8

(a) Give the geometrical interpretation of Rolle's theorem.

(b) Show that the function defined by

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable in any interval.

(c) Investigate the continuity at (1, 2)

$$f(x, y) = \begin{cases} x^2 + 2y & (x, y) \neq (1, 2) \\ 0 & (x, y) = (1, 2) \end{cases}$$

(d) Define beta integrals of 1st and 2nd kinds.

3. Answer any *three* questions : 5×3=15

(a) If

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 \quad ; & (x, y) = 0 \end{cases}$$

show that both the partial derivatives exist at (0, 0) but the function is not continuous thereat.

(b) Test for uniform convergence, the sequence $\{f_n\}$, where

$$f_n(x) = \frac{nx}{1 + n^2 x^2}$$

for all real x.

(c) Show that

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \beta(m, n)$$

(d) State and prove the Taylor's expansion for two variables.

(e) If

$$u_1 = \frac{x_2 x_3}{x_1}, \; u_2 = \frac{x_1 x_3}{x_2}; \; u_3 = \frac{x_1 x_2}{x^3}$$

prove that $J(u_1, u_2, u_3) = 4$.

4. Answer any *three* questions : 10×3=30

(a) *(i)* Show that if f is bounded and integrable on $[a, b]$, then $|f|$ is also bounded and integrable on $[a, b]$. 5

(ii) Prove that

$$\int_{-1}^{1} |x| dx = \lim_{\mu(p) \to 0} S(p, f) = 1$$ 5

(b) *(i)* Show that a function which is differentiable at a point, possesses the first-order partial derivative thereat. 5

(ii) Show that, if

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 \quad ; & (x, y) = (0, 0) \end{cases}$$

then the function possesses partial derivatives at (0, 0). 5