**2013**

PHYSICS

( Major )

Paper : 5.1

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**( Mathematical Methods )**

( *Marks* : 30 )

**1.** For $z = \dfrac{1+i}{(2-3i)^2}$

*(a)* find Re $z$ and Im $z$

*(b)* find Mod $z$

*(c)* find arg $z$

*(d)* give the graphical representation of $z$.

1×4=4

**2.** *(a)* Find the roots of $\sqrt[3]{i}$ and locate them graphically. 2

*(b)* Define equivalent contour. 2

*Or*

Find the value of $(1+i)^5$.

**3.** *(a)* Determine if the following functions are analytic : 4

*(i)* $\dfrac{1+z}{1-z}$

*(ii)* $e^{iz}$

*(b)* Using Cauchy's integral formula, find the value of the integral

$$I = \oint \frac{z^2}{z^2-1}\,dz$$

around the unit circle at *(i)* $z=1$, *(ii)* $z=-1$. 4

*Or*

Find the Taylor series expansion about the origin for $f(z) = \dfrac{1}{(1-z)^m}$ and hence find the series for $\phi(z) = \dfrac{1}{1-z}$.





**4.** *(a)* Evaluate the integral

$$I = \int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$$ 7

*(b)* For a function $f(z)$ which has a pole of order $m$ at $z = z_0$, show that the residue of the function at that singular point is

$$a_{-1} = \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}[(z-z_0)^m f(z)]_{z=z_0}$$

hence find the singular points and calculate the residues for $f(z) = \dfrac{e^z}{(z-2)^3}$.

5+2=7

*Or*

State and derive the Cauchy-Riemann conditions and then use them to compute the first derivative of $f(z) = e^z$. 5+2=7

GROUP—B

**( Classical Mechanics )**

( *Marks* : 30 )

**5.** Answer the following questions : 1×4=4

*(a)* State Hamilton's principle.

*(b)* State one advantage of Lagrangian formulation over Newtonian formulation.

(c) A system of ten particles has five holonomic constants. How many generalised coordinates are required to describe the motion?

(d) What is virtual work? State the principle of virtual work.

**6.** (a) A Lagrangian is given by

$$L = \frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2$$

where $\alpha$ and $\beta$ are constants. Find the Hamiltonian of the system. 2

(b) A particle moves in a circular orbit obeying inverse square law. Show that its angular momentum varies as the square root of its radius. 2

*Or*

What is a cyclic coordinate? Show that a cyclic coordinate in Lagrangian is also a cyclic coordinate in Hamiltonian.

**7.** Answer any *two* of the following questions : 4×2=8

(a) Establish d'Alembert's principle.

(b) Set up the Lagrangian of a compound pendulum and obtain its equation of motion.





(c) Deduce an expression of reduced mass of a two-body central force problem.

**8.** (a) (i) Set up the differential equation of the orbit of a particle under the influence of a central force $F(r)$.

(ii) Show that if the position vector of a particle is given by $r = a\sin\theta$, then $F(r) \propto \frac{1}{r^5}$. 4+3=7

(b) If the Lagrangian of a conservative system does not contain time explicitly, show that the total energy of the system is conserved. Using Lagrange's equation, show that $F_x = -\frac{\partial V}{\partial x}$. 5+2=7

*Or*

Define Hamiltonian of a system and then derive Hamilton's canonical equations. 2+5=7

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(c) The operating voltage of an X-ray tube is increased by 10%. The minimum wavelength of the X-rays emitted

(i) remains the same

(ii) increases by 10%

(iii) decreases by 10%

(iv) initially increases by 10% and then decreases by 10%

(d) Which of the following is *not* true about Raman effect?

(i) Collision between photon and molecules takes place

(ii) Electrons may be ejected from molecules

(iii) Frequency shift depends on scattering molecules

(iv) Frequency shift does not depend on the frequency of the incident radiation

(e) Which of the following is *not* true about Stark effect?

(i) Electric field splits spectral lines

(ii) The effect can be observed in Balmer series of hydrogen spectra

(iii) Frequency shift of component lines is independent of the strength of the electric field

(iv) Magnetic field is not needed





(f) Which element has a hydrogen-like spectrum with spectral lines having wavelength nine times smaller than the hydrogen lines?

(i) $He^{+}$

(ii) $Li^{++}$

(iii) $Be^{+++}$

(iv) $B^{++++}$

(g) The Landé $g$ factor of the $3^2S_{1/2}$ energy level is

(i) $\frac{1}{2}$

(ii) 2

(iii) $\frac{2}{3}$

(iv) 3

**2.** Answer any *four* questions from the following : 2×4=8

(a) An element emits a discrete line spectrum. At very high temperatures the same element emits a continuous spectrum. Briefly explain, why.

**2013**

PHYSICS

( Major )

Paper : 5.2

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Choose the correct option : 1×7=7

*(a)* A hydrogen atom in the ground state absorbs an electromagnetic wave of energy 18 eV and emits its electron. The kinetic energy of the electron is

*(i)* 31·6 eV *(ii)* 18 eV

*(iii)* 13·6 eV *(iv)* 4·4 eV

*(b)* All ions entering the Bainbridge mass spectrograph have the same

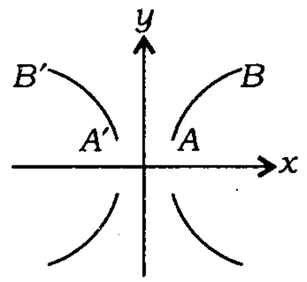
*(i)* mass

*(ii)* velocity

*(iii)* momentum

*(iv)* charge

(b) The photographic plate in Thomson's experiment on positive rays shows the parabolic trace $AB$ of a particular type of ion. A similar trace $A'B'$ on the other side of the $y$-axis is seen on the same plate. Briefly state how $A'B'$ is formed.




(c) The energy levels of a gas are respectively $-20$ eV, $-10$ eV, $-4$ eV, $-2$ eV, $-0\cdot5$ eV .... The gas is irradiated with electromagnetic radiations with wavelengths 860 nm, 743 nm, 621 nm, 555 nm, 511 nm, 207 nm and 194 nm. Which wavelengths will be missing in the transmitted radiations? Take $hc = 1242$ eV nm.

(d) The quantity $(m_L + 2m_S)$ is called the strong field quantum number. Show that this may be used to explain normal Zeeman effect.

(e) The first spectral line of a gas is found to be 414 nm. What is the first excitation potential of the gas?

(f) Mention what happens to the $\vec{L}$ and the $\vec{S}$ vectors of an atom when it is placed in a *(i)* weak and *(ii)* strong external magnetic fields.





**3.** Answer question *(a)* and any *two* from *(b)*, *(c)* and *(d)* : 5×3=15

(a) Inside the apparatus used in the Stern-Gerlach experiment a stream of atoms, each with mass $2\times10^{-22}$ g, is shot with a velocity of $0\cdot6$ km s$^{-1}$ along the $x$-axis into a magnetic field which varies as $\frac{dB_Z}{dZ} = 3\cdot6$ T per mm. The magnetic moment of each atom is 1 Bohr magneton. If the magnetic field extends from $x_1 = 5$ cm to $x_2 = 7$ cm, find the separation of the two beams at $x_2$.

(b) How did Sommerfeld try to improve upon Bohr's model of the atom? What was the result of the attempt? Mention two achievements of the vector atom model.

(c) Using Moseley's law discuss briefly how the X-ray spectral lines are characteristics of the elements from which they are emitted.

(d) Write short note on any *one* of the following :

*(i)* Anomalous Zeeman effect

*(ii)* Paschen-Back effect

*(iii)* Alkali spectra

**4.** Answer questions *(a)* and *(b)*, and any *one* from *(c)*, *(d)* and *(e)* : 10×3=30

*(a)* Deduce Rutherford's formula on scattering of alpha particles by a nucleus. Calculate the distance of closest approach of 1·6 MeV alpha particles scattered by a silver nucleus ($Z = 47$). Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ SI units. 7+3=10

*(b)* Briefly explain the origin of magnetism in an atom. How does normal Zeeman effect confirm that atom behaves as a magnetic dipole? Distinguish between anomalous Zeeman effect and Paschen-Back effect. When a source emitting a single-spectral line with wavelength 600 nm is placed in a magnetic field of $4\pi\times10^{-2}$ T, the Zeeman shift of the two components is found to be 0·0216 Å. Use the data to calculate the value of $e/m$ of electron. 2+2+2+4=10

*(c)* Draw a neat and labelled diagram of the apparatus used by Thomson for determination of $q/M$ of positive rays. Show mathematically, that positive ions with the same $q/M$ value trace out a parabola. Explain how the mass of an isotope can be determined from the parabolic traces. 2+6+2=10





*(d)* What is Rayleigh scattering? Why fog lamps in cars use yellow colour light? Mention two differences between Rayleigh scattering and Compton scattering. A photon of energy $h\nu$ undergoes Compton scattering due to an electron by an angle θ. Show that the energy of the scattered photon is

$$\frac{h\nu}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)}$$

where $m_0$ is the rest mass of the electron and $c$ is the speed of light in vacuum. 1+2+2+5=10

*(e)* Describe the working of a Bainbridge mass spectrograph and draw a neat diagram of the apparatus. Explain how it can be used to measure the masses of isotopes. Mention two advantages of Bainbridge mass spectrograph over Aston's mass spectrograph. It is seen that three traces are formed on the photographic plate in a Bainbridge mass spectrograph. The traces are at 2 mm, 2·8 cm and 3·2 cm from the edge of the plate. If the 2 mm trace is due to hydrogen, then identify the other two elements. 4+1+2+3=10

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# 2013

PHYSICS

( Major )

Paper : 5.3

**( Quantum Mechanics and Astrophysics )**

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**( Quantum Mechanics )**

( *Marks* : 40 )

**1.** Choose the correct answer from the given alternatives (any *four*) : 1×4=4

*(a)* The magnitude of Compton shift is maximum when the angle of scattering is

*(i)* 0° *(ii)* π

*(iii)* π /2 *(iv)* 45°

(b) In view of the uncertainty principle, the radiation emitted by an atom can have

(i) a definite frequency

(ii) a band of frequencies

(iii) a definite phase

(iv) a definite wavelength

(c) Schrödinger one-dimensional wave equation is

(i) first-order in $x$ and second-order in $t$

(ii) second-order in $x$ and first-order in $t$

(iii) first-order in both $x$ and $t$

(iv) second-order in both $x$ and $t$

(d) If $\hat{A} = \frac{d^2}{dx^2}$ and $\psi(x) = ae^{-2x}$, then the eigenvalue of $\psi(x)$ is

(i) 2 (ii) 4

(iii) $2a$ (iv) $a$

(e) If $R$ and $T$ be the reflection and transmission probabilities of a particle through a potential step, then

(i) $R - T$ = constant

(ii) $R + T = 0$

(iii) $R + T = 1$

(iv) $R + T = \infty$





**2.** Answer any *three* questions : 2×3=6

(a) What was the reason for the failure to explain the blackbody radiation spectra? How did Planck overcome this?

(b) What voltage must be applied to an electron microscope to produce electrons of wavelength 0·40 Å?

(c) "Quantum mechanics is probabilistic whereas classical mechanics is deterministic." Discuss.

(d) Suppose the wavelength of incident light in a photoelectric experiment changes from 300 nm to 400 nm. Find the corresponding change in the stopping potential.

(e) Write down Schrödinger's time-dependent and time-independent equations for a particle of mass $m$.

**3.** Answer any *two* questions : 5×2=10

(a) Give the mathematical statement of the uncertainty principle explaining the terms involved. A proton is confined to a nucleus of radius $5 \times 10^{-15}$ m. Calculate the minimum uncertainty in its momentum. What is the minimum uncertainty in the kinetic energy of the proton? Given, mass of the proton $m_p = 1 \cdot 67 \times 10^{-27}$ kg. 1+2+2=5

(b) What do you mean by expectation value of a dynamical variable? Find the expectation values $\langle p \rangle$ and $\langle p^2 \rangle$ for the wave function

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \quad \text{for } 0 < |x| < L$$
$$= 0 \quad \text{for } |x| > L \qquad 1+2+2=5$$

(c) What is tunneling? Is there any similar process in classical mechanics analogous to tunneling in quantum mechanics? Discuss. 3+2=5

(d) Define phase velocity and group velocity. Deduce the relation between phase velocity and group velocity for the de Broglie waves. Which of these two is associated with particle velocity? 2+2+1=5

**4.** What is a one-dimensional potential step? A particle of mass $m$ is moving in a one-dimensional potential given by

$$V = 0 \quad \text{for } x < 0$$
$$= V_0 \quad \text{for } x \geq 0$$

If the energy $E$ of the incident particle is greater than $V_0$, then calculate the coefficient of reflection and transmission. 2+8=10





*Or*

Discuss briefly the experiment of Davisson and Germer pointing out clearly how it demonstrate the existence of de Broglie matter waves. What is the wave-particle duality? 8+2=10

**5.** What is the need for normalization of a wave function? Calculate the normalization constant of a wave function (at $t = 0$) given by

$$\psi(x) = ae^{-(a^2x^2/2)} \cdot e^{ikx}$$

known as the Gaussian wave packet. Hence determine the probability density of the wave function. 2+8=10

*Or*

What are Hermitian operators? Give two examples. What are conjugate variables in quantum mechanics? Give an example of any one pair of conjugate variables and obtain their commutation relations. 2+2+6=10

GROUP—B

**( Astrophysics )**

( *Marks* : 20 )

**6.** *(a)* Answer any *one* question : 1

*(i)* What is a celestial sphere?

*(ii)* What is the declination of the north celestial pole?

*(b)* Answer any *two* questions : 2×2=4

*(i)* Define ecliptic. What are vernal and autumnal equinoxes?

*(ii)* What is the difference between sidereal time and solar time?

*(iii)* Explain the signs of zodiac.

**7.** Answer any *one* question : 5

*(a)* The apparent magnitude of a star is found to be +3·3 and its parallax is 0·005″. Find its absolute magnitude.

*(b)* What are Hertzsprung-Russell (H-R) diagram? What is the meaning and importance of the term main sequence in such diagrams?





**8.** What is the basis of spectral classification of stars? Enumerate the special features of the Harvard spectral sequence. 4+6=10

*Or*

Write down the sequence of events leading to the formation of a protostar. When does a protostar become a star? Describe briefly the occurrence of helium flash. 5+2+3=10

[ The following data can be used when required :

Charge of electron $e = 1 \cdot 6 \times 10^{-19}$ C

$c = 3 \times 10^{8}$ m/s, $h = 6 \cdot 6 \times 10^{-34}$ J-s

$m_e = 9 \cdot 1 \times 10^{-31}$ kg, $m_p = 1 \cdot 67 \times 10^{-27}$ kg ]

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**8.** Answer *either (a)* and *(b) or (c)* and *(d)* of the following questions : 5+5=10

*(a)* A sinusoidal carrier wave of frequency 10 MHz and power 500 watt is amplitude modulated by a modulating signal of frequency 100 kHz to a depth of 80%. Calculate *(i)* modulation index, *(ii)* power in each sideband, *(iii)* total power of the modulated signal and *(iv)* frequencies of LSB and USB.

*(b)* Draw the circuit diagram of a square law amplitude modulator and give the mathematical analysis of generation of different sidebands.

*Or*

*(c)* What is the frequency range of radio waves? Discuss different ways of radio wave propagation giving appropriate frequency range and nature of modulation and approximate distance between the transmitting and receiving antenna.

*(d)* What is a time base of a CRO? Explain how the phase difference between two voltages can be measured using Lissajous figure produced in a CRO. What is a bus of a microprocessor? Discuss briefly about major buses of a microprocessor. Which bus is bidirectional?

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# 2013

## PHYSICS

( Major )

Paper : 5.4

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Answer the following questions very briefly : 1×7=7

*(a)* Explain how the reverse saturation current depends on temperature.

*(b)* How does a shunt capacitor work as a filter?

*(c)* What are the basic units of a regulated power supply system?

*(d)* What types of biasing are necessary for 'transistor' action?

*(e)* Give the statement of maximum power transfer theorem when an energy source delivers power to a load impedance.

(f) Give examples of AF and RF oscillators.

(g) Express common-mode rejection ratio in dB.

**2.** Answer the following questions : 2×4=8

(a) Draw the d.c. and a.c. load lines along with the CE mode output characteristics of a transistor and locate the *Q*-point on them.

(b) Draw the block diagrams of voltage series feedback and current shunt feedback.

(c) Compare amplitude modulation with frequency modulation.

(d) Draw the logic symbol diagram to convert *J-K* flip-flop to *D* and *T* flip-flops.

**3.** Draw the schematic energy-band diagram of an unbiased and a forward biased *P-N* junction diode. A small portion of the peak of the negative half cycle of an a.c. signal voltage is to be removed. Draw a circuit diagram for this purpose and explain its working giving the waveform of the input and output signals. 5

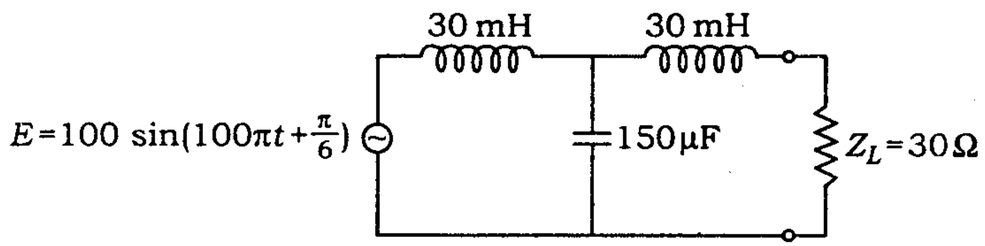
*Or*

Draw the circuit diagram of a *P-N* junction diode full-wave rectifier. Derive the expressions for its efficiency and ripple factor.





**4.** Calculate Thevenin voltage, Thevenin impedance and load current in case of the following circuit : 5




**5.** Draw the circuit diagram of a tuned collector transistor oscillator. Derive an expression for condition of sustained oscillation in terms of circuit elements using its *h*-parameter equivalent circuit. 5

*Or*

Draw the circuit diagram of an astable multivibrator and explain its working giving waveform of the output signal across the two transistors.

**6.** Answer *either (a)* and *(b) or (c)* and *(d)* of the following questions : 5+5=10

(a) What is the need of stabilisation of *Q*-point of a transistor in operation? Define different stability factors. Draw the circuit diagram of self-bias with potential divider arrangement for an *N-P-N* transistor amplifier and explain qualitatively how the biasing and stability conditions are achieved.

(b) A two-stage $R$-$C$ coupled amplifier uses two identical transistors having $h$-parameters $h_{ie} = 1\ k\Omega$, $h_{fe} = 100$ and negligible $h_{oe}$ and $h_{re}$. If the load resistance $R_L = 3\ k\Omega$, coupling capacitor $C_c = 2\ \mu F$ and effective shunt capacitance across the load for high frequency range is $0 \cdot 02\ \mu F$, calculate the following for the 1st stage :

(i) Mid-frequency voltage gain

(ii) Coordinates of lower and upper 3 dB points in the frequency response curve

(iii) Bandwidth

*Or*

(c) How can a transistor be considered as a two-port of four-terminal device? What are the variables related to input and output ports in case of a transistor? Establish the relations of $h$-parameters with these variables for small input a.c. signal and hence draw the $h$-parameter a.c. equivalent circuit.

(d) What is the basic principle of power amplifier? Draw the circuit diagram of a class-$B$ push-pull power amplifier using power transistors and derive an expression for its efficiency. What is the percentage of maximum efficiency?





**7.** Answer *either (a)* and *(b) or (c)* and *(d)* of the following questions : 5+5=10

(a) Draw the circuit diagram of a two-stage direct coupled amplifier using transistors. Derive expressions for voltage gain for different frequency range using $h$-parameter equivalent circuit.

(b) (i) Convert binary 110·111 to its decimal equivalent.

(ii) Convert decimal 4021·25 to its binary equivalent.

(iii) Add :

$$(101)_2 + (111)_2 + (100)_2$$

(iv) Subtract :

$$(11011)_2 - (100101)_2$$

*Or*

(c) Write down the characteristics of an ideal OP-AMP. Draw the circuit diagram of an OP-AMP in inverting mode and derive the expression for its closed-loop gain. Explain briefly the operation of an OP-AMP as integrator with circuit diagram.

(d) What is a clock pulse used in synchronous sequential circuits? Draw a simple circuit diagram which can convert a clock pulse to a spike. What is the basic difference between a latch and a flip-flop from the viewpoint of triggering mechanism? What is race-around condition? Draw circuit diagram of a flip-flop where this condition is eliminated and briefly describe its working.