(b) If A be an $n \times n$ matrix, show that the rank of Adj A is n, 1 or 0 according as the rank of A is n, $n-1$ or less than $n-1$.

5. Answer *(a)* or *(b)* : 10

(a) If A, B are two n rowed square matrices, then show that

$$\text{Rank } (AB) \geq (\text{Rank } A) + (\text{Rank } B) - n$$

(b) Show that the rank of the product of two matrices cannot exceed the rank of either matrix.

6. Answer *(a)* or *(b)* : 10

(a) Solve completely the system of equations

$$2x + 6y - 4z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

(b) Find a solution to the system of linear equations

$$2x + 7z = 4$$
$$3x + 3y + 6z = 3$$
$$2x + 2y + 4z = 2$$

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2014

STATISTICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×7=7

(a) When is a matrix said to be orthogonal?

(b) State the condition under which a system of non-homogenous linear equations will have no solution.

(c) Define a positive definite quadratic form.

(d) If the rank of matrix A is r, what is the rank of cA (where c is a constant)?

(e) Write true or false :

The interchange of rows of a matrix alters the rank of the matrix.

(f) Find the rank of

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(g) Are the ranks of the following matrices equal? Write Yes or No.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 0 \\ 0 & 2 & 3 & 4 & 1 & 0 \\ 0 & 3 & 2 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Answer the following : 2×4=8

(a) If matrices A and B are such that $AB = A$ and $BA = B$, show that B is idempotent.

(b) Show that the inverse of a non-singular matrix is unique.

(c) Show that if P and Q are orthogonal matrices, then so is PQ.

(d) If matrices A and B commute, show that A^{-1} and B^{-1} also commute.

3. Answer any *three* of the following : 5×3=15

(a) If A and B are two square matrices of order $n \times n$ and $AB = I$, then show that $BA = I$.

(b) If A is an n rowed non-singular matrix, X be an $n \times 1$ matrix, B be an $n \times 1$ matrix, then show that the system of equations $AX = B$ has a unique solution.

(c) If A and B are square matrices of the same order, then show that

$$\text{adj}(AB) = (\text{adj } A)(\text{adj } B)$$

(d) Find the matrices R and S such that

$$R \begin{bmatrix} 2 & 2 & -6 \\ -1 & 2 & 2 \end{bmatrix} S$$

is in normal form.

(e) Write the matrix and find the rank of each of the following quadratic forms :

(i) $x_1^2 - 2x_1x_2 + 2x_2^2$

(ii) $4x_1^2 + x_2^2 - 8x_3^2 + 4x_1x_2 - 4x_1x_3 + 8x_2x_3$

4. Answer *(a)* or *(b)* : 10

(a) If A is an $n \times n$ matrix, prove in detail that the determinant of the adjoint of A is $|A|^{n-1}$.

(b) Obtain Poisson distribution as a limiting case of negative binomial distribution.

5. Answer *either (a) or (b)* : 10

(a) If $X_1, X_2, \cdots, X_n$ are n-independent observations from a distribution with p.d.f. $f(x)$ and distribution function $F(x)$, then find the p.d.f. of—

(i) $Y = \max(X_1, X_2, \cdots, X_n)$;

(ii) $Z = \min(X_1, X_2, \cdots, X_n)$.

(b) If X and Y are independent gamma variates, then obtain the distribution of $\frac{X}{Y}$.

6. Answer *either (a) or (b)* : 10

(a) Let X follow the standard Cauchy distribution. Show that X^2 follows

$$\beta_2\left(\frac{1}{2}, \frac{1}{2}\right)$$

(b) Show that for a normal distribution with mean μ and variance σ^2, the central moments satisfy the relation

$$\mu_{2n} = \sigma^2 (2n-1)\mu_{2n-2}$$

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STATISTICS

(Major)

Paper : 3.2

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) If X is a Bernoulli random variable, find $\text{Var}(X)$.

(b) Given that X has a Poisson distribution with variance 0·5. Calculate $P(X = 3)$.

(c) Let $X \sim N(\mu, \sigma^2)$. State what is $E(X-\mu)^7$.

(d) If X follows Cauchy distribution, state what is μ'_3.

(e) If the mean of gamma distribution is 3, what is its variance?

(f) If $X \sim \text{Bin}(m, p)$ and $Y \sim \text{Bin}(n, p)$, where X and Y are independent, what is the distribution of $X + Y$?

(g) If $X \sim N(\mu, \sigma^2)$, what is the distribution of $Z = \frac{1}{2}\left(\frac{X - \mu}{\sigma}\right)$?

2. Answer the following questions : 2×4=8

(a) In an examination, marks obtained by students in Mathematics, Physics and Chemistry are distributed normally with means 50, 52 and 48 with standard deviations 15, 12 and 16 respectively. Find the probability of scoring total marks of 150 or more.

(b) State the conditions under which the distribution of number of printing mistakes in a page of a book follows Poisson distribution.

(c) State the p.d.f. of Weibull distribution and also state the condition under which the Weibull p.d.f. reduces to exponential distribution.

(d) Give an interpretation of the p.m.f. of geometric distribution.

3. Answer any *three* of the following : 5×3=15

(a) Prove that the recurrence formulae for negative binomial distribution is

$$f(x+1; r, p) = \frac{x+r}{x+1} q f(x, r)$$

(b) If X and Y are two independent random variables each representing the number of failures preceding the first success in a sequence of Bernoulli trials with p as the prob of success in a single trial and q as the prob of failure, then show that

$$P(X = Y) = \frac{p}{1+q}$$

(c) Obtain the mean deviation about mean of uniform distribution $u(a, b)$.

(d) If $X \sim N(\mu, \sigma^2)$, find the m.g.f. of X.

(e) Prove that binomial distribution tends to normal distribution under certain conditions.

4. Answer *either* (a) *or* (b) : 10

(a) Prove that for the binomial distribution, the following relation holds good :

$$\mu_{r+1} = pq\left(\frac{d\mu r}{dp} + nr\mu_{r-1}\right)$$