2012

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Choose the correct option/Answer the following : 1×3=3

(a) What is the modulus of the determinant of a unitary matrix?

(i) 1

(ii) 0

(iii) −1

(iv) None of these

(b) What is a Hermitian matrix?

(c) What is a skew-symmetric matrix?

2. Define conjugate transpose of a matrix. Show that

$$(AB)^+ = B^+ A^+$$ 1+1=2

3. Answer any *two* questions out of *(a)*, *(b)* and *(c)* :

(a) *(i)* For three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

prove that $\sigma_i \sigma_j = i\sigma_k$, where i, j, k are cyclic permutations of indices. 3

(ii) Show that modulus of each eigenvalue of a unitary matrix is unity. 2

(b) *(i)* Verify that

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is an orthogonal matrix. 2

(ii) Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1}$$ 3

(c) What is a frame of reference? A reference frame a rotates with respect to another frame b with uniform angular velocity $\vec{\omega}$. If the position, velocity and acceleration of a particle in frame a are represented by $\vec{r}$, $\vec{V}_a$ and $\vec{f}_a$ respectively, show that the acceleration of that particle in frame b is given by $\vec{f}_b$, where

$$\vec{f}_b = \vec{f}_a + 2\vec{\omega} \times \vec{V}_a + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$ 5

4. Answer *either (a)* and *(b)* *or (c)* and *(d)* :

Either

(a) State Cayley-Hamilton theorem. Obtain the characteristic equation of a matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

and verify Cayley-Hamilton theorem. 5

(b) Find the mutually perpendicular eigenvectors of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ 5

Or

(c) Show that the trace of a product of two matrices is independent of the order of multiplication. Also show that eigenvalues of a Hermitian matrix are all real and its eigenvectors corresponding to two distinct eigenvalues are orthogonal. 2+3=5

(d) For the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

determine a matrix P such that $P^{-1}AP$ is a diagonal matrix. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct option/Answer the following : 1×4=4

(a) Electric field vector $\vec{E}$ is

(i) rotational

(ii) irrotational

(b) What do you understand by electrical octupole?

(c) What is meant by electrical image?

(d) Define electrical susceptibility.

6. Answer the following questions : 2×3=6

(a) The electric field due to a short dipole at a point distant 1 cm from it on its perpendicular bisector is $1 \cdot 5 \times 10^{-11}$ volt/m. Find the dipole moment.

(b) Write down Poisson's equation.

(c) What is a polar molecule? Define molecular polarizability.

7. Write down the integral as well as differential form of Gauss' law. Use this law to show that the expression for field strength at a distance r due to an infinite line charge is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

where λ is linear charge density and r is the distance of the external point from the line charge. 1+1+3=5

Or

Show that the interaction energy of two dipoles of moments $\vec{p}_1$ and $\vec{p}_2$ is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}_1 \cdot \vec{p}_2}{r^3} - \frac{3}{r^5}(\vec{p}_1 \cdot \vec{r})(\vec{p}_2 \cdot \vec{r})\right]$$

where $\vec{r}$ is the radius vector joining the centres of the two dipoles. Hence derive the torque acting on any dipole due to the field of another dipole. 4+1=5

8. Answer any *two* questions out of *(a)*, *(b)*, *(c)* and *(d)* :

(a) *(i)* Show that the electric field due to an electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. 5

(ii) Show that the energy density of electrostatic field in free space is given by

$$U = \frac{1}{2}\varepsilon_0 E^2$$

where the symbols have got their usual meanings. 5

(b) (i) State and prove the uniqueness theorem regarding solutions to Laplace's equation. 1+4=5

(ii) Use Laplace's equation to find potential inside spherical capacitor. 5

(c) A point charge is situated near an infinite plane earthed conductor. Apply the method of electrical image to calculate—

(i) surface charge density induced on the plane;

(ii) the force between the plane and the charge.

An electron is at a distance 10 Å from an infinite plane conductor. Calculate the force experienced by the electron and the work done in moving it to infinite distance away from the conductor. 4+3+3=10

(d) (i) A spherical cavity is cut in a dielectric medium. Show that

$$\vec{E}_{\text{eff}} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$$

where the symbols have got their usual meanings. 3

(ii) Deduce Clausius-Mosotti relation. Point out its limitation. 6+1=7

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2012

PHYSICS

(Major)

Paper : 3.2

(Current Electricity and Magnetostatics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is the dimension of capacitance in terms of the fundamental quantities (M, L, T, I)?

(b) Write down the differential form of Faraday's law of electromagnetic induction.

(c) What is the SI unit of thermoelectric power?

(d) Show that when a voltage of sinusoidal waveform is applied across a capacitor, the current passing through it leads the voltage by 90°.

(e) Can a charged particle at rest be accelerated by applying a magnetic field? Justify your answer.

(f) Write down the Maxwell's equation of electromagnetism which shows that magnetic monopole does not exist.

(g) If a sinusoidal voltage is applied to a series *L-C-R* circuit, under what condition the circuit becomes purely resistive?

2. Answer the following questions : 2×4=8

(a) A d.c. voltage V is suddenly applied to a series *R-C* circuit. Calculate the time it takes in charging the capacitor to 0·993 V. (Given $R = 5\ k\Omega$ and $C = 2\ \mu F$)

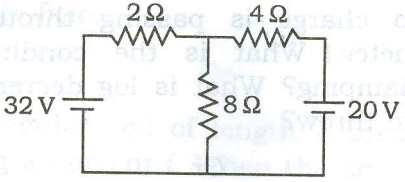
(b) Draw the circuit diagram of Kelvin's double bridge for low resistance measurement.

(c) An air-cored solenoid has a diameter of 2·5 cm and 500 turns wound over a length of 30 cm. Calculate the self-inductance of the solenoid. (Given permeability of air $\mu = 4\pi \times 10^{-7}$ H/m)

(d) Two parallel long straight wires at a distance 1 m apart placed in air carry equal currents $i = 5$ A in the same direction. Find the magnitude of force per unit length of the wires. (Given permeability of air $\mu = 4\pi \times 10^{-7}$ H/m)

3. Answer any *three* of the following questions : 5×3=15

(a) In the circuit given below, find the current and voltage drop across each resistor :



(b) A d.c. voltage of 80 V is switched on to a circuit containing a resistance of 5 Ω in series with an inductance of 20 H. Calculate the rate of growth of current at the instant when the current is *(i)* 6 A and *(ii)* 16 A.

(c) The e.m.f. equation of a Cu-Ni thermocouple (μV) is $E = 16 \cdot 34t - 0 \cdot 021t^2$, where t is the temperature difference between hot and cold junctions and is in °C. Calculate the thermoelectric power and Peltier coefficient if the temperature of the hot junction is 100 °C and cold junction is 0 °C.

(d) Starting from Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

show that $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, where $\vec{J}$ is the current density.

(e) Find the magnetic field at the centre of a circular coil carrying a steady current.

4. Establish the differential equation for a moving-coil ballistic galvanometer and find an expression for instantaneous deflection θ, when no charge is passing through the galvanometer. What is the condition for critical damping? What is log decrement in a ballistic throw? 7+2+1=10

Or

What are meant by 'self' and 'mutual' inductances? Find an expression for self-inductance of an air-cored long solenoid of radius a, length l and having N number of turns in it. 3+7=10

5. A series combination of an inductance L and a resistance R is connected in parallel with a lossless capacitor of capacitance C. A sinusoidal e.m.f. of amplitude V_0 and angular frequency ω is applied across the circuit. Find the resonant frequency and the impedance at resonance. Why is such a circuit known as rejector circuit? 6+2+2=10

Or

A d.c. e.m.f. E is suddenly applied to a circuit consisting of a resistance R, an inductance L and a capacitance C in series. Investigate the growth of charge in the circuit. Discuss the conditions for non-oscillatory and oscillatory growth of charge. 6+4=10

6. Differentiate between magnetic vector potential and magnetic scalar potential. Determine the magnetic vector potential at a distance r from a very long thin straight wire carrying a current I. Hence find the corresponding magnetic field. 3+5+2=10

Or

A rectangular coil of length l, breadth b is carrying a current i. When the coil is placed in a uniform magnetic field $\vec{B}$, establish that the potential energy of the coil is $V = -\vec{m} \cdot \vec{B}$, where $\vec{m}$ is the dipole moment of the current carrying coil. Why can the current carrying coil be considered as a magnetic dipole? 8+2=10

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