(b) For what values of η, the equations

$$x+y+z=1$$
$$x+2y+4z=\eta$$
$$x+4y+10z=\eta^2$$

have a solution? Solve them completely in each case.

★★★

2 0 1 7

MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) What is the order of A_m, alternative group of degree n?

(b) Is generator of a cyclic group always unique?

(c) Does the set of all odd integers form a group with respect to addition?

(d) Define Hermitian matrix.

(e) What is normal form of a matrix?

(f) What is the rank of a matrix, where every element of the matrix is unity?

(g) If in a square matrix A, $|A|=0$, then what is the value of $|\text{adj } A|$?

(h) Find the amplitude of the complex number $-1-i$.

(i) What is the period of $\sinh x$?

(j) State Gregory series.

2. Give the answer of the following questions : 2×5=10

(a) Can a non-Abelian group have an Abelian subgroup? Justify your answer.

(b) Express the following matrix as a sum of symmetric and skew-symmetric matrix :

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

(c) Let A and B be two square matrices of order n. If $AB = 1$, then prove that $BA = 1$.

(d) If the matrices A and B commute, then show that A^{-1} and B^{-1} also commute.

(e) If

$$x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$$

then prove that $x_1 x_2 x_3 \cdots \infty = \cos\pi$.

3. Answer the following questions : 5×2=10

(a) Prove that every group of prime order is cyclic.

(b) Prove that i^i is completely real. Find its principal value.

Or

Prove that

$$\frac{1}{6}\sin^3 x = \frac{x^3}{\lfloor 3} - \frac{1}{\lfloor 5}(3^2+1)x^5 + \frac{1}{\lfloor 7}(3^4+3^2+1)x^7 + \cdots$$

4. Answer any *two* questions : 5×2=10

(a) If α, β, γ are the roots of the equation

$$x^3 - px^2 + qx - r = 0$$

then find the value of

$$\sum\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$$

in terms of p, q and r.

(b) Find the condition that the cubic

$$x^3 - px^2 + qx - r = 0$$

should have its roots in harmonic progression.

(c) Using Descartes' rule of sign, show that when n is even, the equation $x^n - 1 = 0$ has two real roots 1 and −1 and no other real root, and when n is odd, the only real root is 1.

5. Answer any *one* question : 10

(a) Let A be a non-empty set and let R be an equivalence relation in A. Let a and b be arbitrary elements in A. Then prove that—

(i) $[a] = [b]$, iff $(a, b) \in R$;

(ii) either $[a] = [b]$ or $[a] \cap [b] = \phi$.

(b) Prove that an equivalence relation R in a non-empty set S determines a partition of S and conversely, a partition of S defines an equivalence relation in S.

6. Answer any *one* question : 10

(a) If H is a subgroup of G, then prove that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right coset of H in G.

(b) Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is again a right coset of H in G.

7. Answer any *one* question : 10

(a) Find real and imaginary parts of

$$\sin^{-1}(\cos\theta + i\sin\theta)\ (\theta \in R)$$

(b) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2}\log_e \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

8. Answer any *one* question : 10

(a) If A be any n-square matrix, then show that

$$A(\text{Adj}A) = (\text{Adj}A)A = |A| I_n$$

where I_n is the n-rowed unit matrix. Verify it for the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -3 & -2 \end{bmatrix}$$

(ii) Find the volume of the solid generated by the revolution of the curve $(a-x)y^2 = a^2x$ about its asymptote. 5

(b) (i) Find the asymptotes of the curve

$$x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0$$ 5

(ii) Trace the curve $y = x^3$. 5

7. Answer the following questions :

(a) Show that points of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line. 5

8. Answer either (a) or (b) :

(a) Derive a reduction formula for

$$\int \sin^m x \sin nx\, dx$$

Hence evaluate

$$\int_0^{\pi} \sin^m x \sin nx\, dx$$ 7+3=10

(b) What are the double points? Examine the nature of double points of the curve

$$2(x^3 + y^3) - 3(3x^2 + y^2) + 12x = 4$$ 2+8=10

★ ★ ★

2017

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write the nth derivative of $\sin^3 x$.

(b) If $f(x, y) = 3x^2y + 2xy^2$, find $f_x(1, 2)$.

(c) State Euler's theorem on homogeneous function of degree n for two variables.

(d) Write the subtangent of the curve $y^2 = 4ax$.

(e) Define asymptotes.

(f) Write the value of $\int_{-a}^{a} x^3\sqrt{a^2 - x^2}\, dx$.

(g) Define point of inflexion.

(h) For a pedal curve $p = r\sin\phi$, write the formula for radius of curvature.

(i) Write down the reduction formula for $\int \tan^n x\,dx$

(j) What is a cusp?

2. Answer the following questions : 2×5=10

(a) Find nth derivative of $\frac{1}{a^2 - x^2}$.

(b) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, find $\frac{\partial^2 u}{\partial x \partial y}$

(c) The tangent of the curve $y^2 = 4a\left\{x + \sin\frac{x}{a}\right\}$ at (x_1, y_1) is parallel to x-axis. Show that $\cos(x_1 / a) = -1$

(d) Evaluate $\int_0^{\pi} x \sin x \cos^2 x\,dx$.

(e) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

3. Answer the following questions :

(a) (i) If $u = e^{xyz}$, show that
$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$ 3

(ii) Find the pedal equation of the curve
$$x^2 + y^2 = 2ax$$ 2

(b) Derive a reduction formula for $\int \cos^n x\,dx$. 5

4. Answer either (a) or (b) :

(a) (i) Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on
$$x^2y^2 = x^2 - y^2$$ 5

(ii) Evaluate $\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$. 5

(b) (i) Evaluate $\int \frac{dx}{3 + 5\cos x}$. 5

(ii) Evaluate $\int \sqrt{\frac{x-3}{x-4}}\,dx$. 5

5. Answer the following questions :

(a) If $y = [x + \sqrt{1+x^2}]^m$, find the nth derivative of y for $x = 0$. 5

(b) Find the perimeter of the circle
$$x^2 + y^2 = a^2$$ 5

6. Answer either (a) or (b) :

(a) (i) If $u = x\,\phi(y/x) + \psi(y/x)$, prove that
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 y}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$ 5