Total number of printed pages–7

3 (Sem–4/CBCS) STA HC 1

2024

STATISTICS

(Honours Core)

Paper : STA–HC–4016

(Statistical Inference)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

   (a) An estimator $\hat{\theta}$ of a parameter θ is said to be unbiased if ______. *(Fill in the blank)*

   (b) The variance S^2 of a sample of size n is a ________ estimator of population variance σ^2. *(Fill in the blank)*

Contd.

(c) Critical region is also known as _____.

(Fill in the blank)

(d) Maximum likelihood estimator is always unbiased. *(Write True or False)*

(e) If β is the probability of type II error, then $(1-\beta)$ is known as _______.

(Fill in the blank)

(f) The choice of one-tailed or two-tailed test depends on ______.

(Fill in the blank)

(g) If MLE (maximum likelihood estimate) exists, it is the most efficient estimator in the class of such estimators.

(State True or False)

2. Answer the following questions briefly : 2×4=8

(a) If T is an unbiased estimator for θ, show that T^2 is a biased estimator for θ^2.

(b) Define best critical region (BCR) for a test.

(c) Estimate θ for the distribution

$$f(x, \theta) = \frac{2}{\theta^2}(\theta - x),\ 0 < x < \theta$$

for sample of size one.

(d) State the asymptotic properties of likelihood ratio (LR) test.

3. Answer ***any three*** of the following questions : 5×3=15

(a) For a random sample $x_1, x_2, \ldots\ldots, x_n$ from $N(\mu, \sigma^2)$, find sufficient estimators for μ and σ^2.

(b) A random sample $x_1, x_2, \ldots\ldots, x_n$ is taken from a normal population with mean 0 and variance σ^2. Examine whether $\sum_{i=1}^{n} \frac{x_i^2}{n}$ is a minimum variance bound (MVB) estimator for σ^2.

(c) Given that T_n is a consistent estimator of $\gamma(\theta)$ and $\psi\{\gamma(\theta)\}$ is a continuous function of $\gamma(\theta)$, prove that $\psi(T_n)$ is a consistent estimator of $\psi\{\gamma(\theta)\}$.

(d) State Neyman-Pearson (NP) lemma and likelihood ratio test. When do the likelihood ratio principle and NP lemma (Neyman-Pearson lemma) give the same test?

(e) Write a note on sequential probability ratio test.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Define critical region. Also explain type I and type II errors. 2+2+2=6

(ii) In testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ for the frequency distribution

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{otherwise} \end{cases}$$

determine the type I and type II errors for a critical region (CR) $0{\cdot}5 \le x$. 4

(b) (i) Given that $x_1, x_2, \ldots\ldots, x_n$ is a random sample drawn from a Poisson population with parameter λ. Test whether $T = \sum_{i=1}^{n} x_i$ is complete. 7

(ii) Write briefly about the method of maximum likelihood estimation (MLE). 3

(c) Define Minimum Variance Unbiased estimator. Prove that MVU estimator is unique.

(d) (i) What are the regularity conditions for Cramer-Rao inequality? 5

(ii) A random sample $x_1, x_2, \ldots\ldots, x_n$ is taken from uniform population with mean zero and variance θ. Find a sufficient estimator for θ. 5

(e) If W be a most powerful critical region of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ then it is necessarily unbiased.

(f) Determine the best critical region for the test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 > \theta_0$ for a normal population $N(\theta, \sigma^2)$, where σ^2 is known.

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