

Total number of printed pages-7

3 (Sem-6/CBCS) MAT HE 3

2025

MATHEMATICS

(Honours Elective)

Paper : MAT-HE-6036

(*Mathematical Modelling*)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$
 - If R is the radius of convergence of any power series then what is the interval of convergence ?
 - Write Legendre's equation of order n .
 - Find the Laplace transform of $F(t) = e^{at}$ for $t \geq 0$.

B01FS 0114

Contd.

(d) State the linearity property of Laplace transform.

(e) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

(f) The important step required for simulation approach in solving a problem is to

(i) test and validate the model

(ii) design the experiment

(iii) conduct the experiment

(iv) All of the above

(g) What do you mean by Simulation Modelling?

2. Answer the following questions : $2 \times 4 = 8$

(a) Show that $x=0$ is an ordinary point of

$$(x^2 - 1)y'' + xy' - y = 0,$$

but $x=1$ is a regular singular point.

(b) If the Laplace transform of the function $f(t)$ is given by $\frac{s+3}{(s+1)(s+2)}$, then find the value of $f(0)$.

(c) Find the inverse Laplace transform of

$$G(s) = \frac{1}{s^2(s-a)}.$$

(d) Why is sensitivity analysis important in linear programming.

3. Answer **any three** questions of the following : $5 \times 3 = 15$

(a) Find the power series solution of the differential equation

$$(x^2 + 1)y'' + xy' - xy = 0.$$

(b) Find the exponents in the possible Frobenius series solution of the equation

$$2x^2(1+x)y'' + 3x(1+x)^3 y' - (1-x^2)y = 0.$$

(c) Evaluate: $2\frac{1}{2} + 2\frac{1}{2} = 5$

$$(i) L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$$

$$(ii) L\left\{\int_0^t ue^{-u} \sin 4u \, du\right\}$$

(d) Solve the following differential equation using Laplace transformation

$$(x-3)y' + 2y = 0.$$

(e) What is meant by random numbers in Monte Carlo simulation? Use linear congruence method to generate 8 random numbers using $a = 5$, $b = 1$ and $c = 8$ and taking initial seed $x_0 = 17$.

$$1+4=5$$

4. Answer **any three** questions of the following: $10 \times 3 = 30$

(a) Find the general solution in powers of x of the equation

$$(x^2 - 4)y'' + 3xy' + y = 0.$$

Then find the particular solution with $y(0) = 4$, $y'(0) = 1$.

(b) Find the Frobenius series solution of

$$2x^2y'' + 3xy' - (x^2 + 1)y = 0.$$

(c) (i) Solve the initial value problem
 $x'' + 4x = \sin 3t, x(0) = 0 = x'(0)$. 5
(ii) Find the inverse Laplace transformation to find $f(t)$ of the function

$$F(s) = \frac{s-2}{s^2 + 5s + 6} \quad 5$$

(d) Using Monte Carlo simulation, write an algorithm to calculate the volume of the sphere

$$x^2 + y^2 + z^2 \leq 1$$

that lies in the first octant $x > 0, y > 0, z > 0$.

(e) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and amount of cargo and varies from 45 to 90 minutes.

Below is given a situation with 5 ships :

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships :	30	15	20	25	120
Unloading time :	40	35	60	45	75

(i) Draw the timeline diagram depicting clearly the situation for each ship, the idle time for the harbor and the waiting time.
(ii) Find the average waiting time for the ships. $8+2=10$