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3 (Sem-6/CBCS) MAT HE 1

2025

MATHEMATICS

(Honours Elective)

Paper : MAT-HE-6016

(Boolean Algebra and Automata Theory)

Full Marks : 80

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Give very short answers to the following :
1×10=10
 - (a) Define an ordered set.
 - (b) Define a poset.
 - (c) When are two elements of a poset called comparable?
 - (d) When does an ordered set become a total ordered set?

- (e) Write the absorption law of lattice.
- (f) When are two lattices called isomorphic?
- (g) Find the minimal and maximal elements of the ordered set $(\{1, 2, 3, 4, 6, 8, 12\}, |)$, where $|$ stands for divisibility.
- (h) Define complement elements in Boolean algebra.
- (i) Write true **or** false: "Every language accepted by a deterministic automaton is accepted by a non-deterministic automaton."
- (j) Draw a state diagram for an automaton which accepts the language expressed by $aa*bb*cc*$.
2. Give answers to the following: $2 \times 5 = 10$
- (a) Prove that every finite lattice is bounded.

- (b) Draw the Hasse diagram for the lattice $(\{1, 3, 6, 12, 24\}, |)$, where $|$ stands for divisibility.
- (c) Draw a diagram for the Boolean expression $(x + y + z)(xy' + x'z)$.
- (d) Define Alphabets in automata theory. Which are commonly used alphabets?
- (e) What is a string in automata theory? Give an example.
3. Give answers to the following: **(any four)**
 $5 \times 4 = 20$
- (a) Let L be a bounded distributive lattice. Show that the complement of L if exists, is unique.
- (b) Let $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and consider the order relation ' \leq ' of divisibility on A . Let $B = P(S)$, the power set of S ; where $S = \{a, b, c\}$ be the ordered set with order relation ' \subseteq '. Show that (A, \leq) and (B, \subseteq) are isomorphic.

(c) Express $(x+y)(x'+z)$ and x in CNF of three variables x, y, z . $2\frac{1}{2}+2\frac{1}{2}=5$

(d) When do you use the Quinn-McCluskey theorem in Boolean algebra? Write the four main steps in the Quinn-McCluskey algorithm. $1+4=5$

(e) Name three types of indirect proofs. Prove that $\sqrt{2}$ is not a rational number. What is the type of proof applied in this context? $1\frac{1}{2}+2\frac{1}{2}+1=5$

(f) What is Deterministic Finite automata? What are the elements of Deterministic Finite automata? $2+3=5$

4. Give answers to the following: (**any four**) $10 \times 4 = 40$

(a) (i) Use Karnaugh maps to find a minimal form for the Boolean function $E(x, y) = x'y' + xy'$. 3

(ii) Show that the set of gates (AND, NOT) is functionally complete. 3

(iii) Construct a logic circuit corresponding to the Boolean function

$f(x, y, z) = xyz' + xy'z + x'yz$. Also simplify and draw a simpler logic circuit. $2+2=4$

(b) Define a partial order relation in a set. Examine whether the following relations satisfy all axioms of a partial order relation. $2+4+4=10$

(i) A relation \sim on the set of real numbers such as $x \sim y$ if and only if $x^3 - 4x \leq y^3 - 4y$.

(ii) A relation \sim on the set R^2 such as $(a, b) \sim (c, d)$ if and only if $|ab| \geq |cd|$.

(c) (i) For any Boolean algebra B , show that $(a+b)(b+c)(c+a) = ab+bc+ca$ for all elements a, b, c of B . 5

(ii) State and prove the De Morgan's laws in Boolean algebra. 5

(d) (i) Express $xy' + y(x' + z)$ in DNF in the variables present. 5

(ii) Express $(x + y' + z)(xy + x'z)$ in CNF in the variables present. 5

(e) Define a complemented lattice. Give an example of a complemented lattice. Show that two bounded lattices L and M are complemented if and only if $L \times M$ is complemented. 1+1+8=10

(f) Show that the mapping $f: B \rightarrow P(A)$ is an isomorphism where B is a Boolean Algebra, $P(A)$ is the power set of the set A of atoms and $f(x) = [a_1, a_2, \dots, a_n]$ where $x = a_1 + a_2 + \dots + a_n$ is the unique representation of $a \in A$ as a sum of atoms.

(g) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design a circuit as simple as you can which will allow current to pass when and only when a proposal is approved.

(h) Prove that a language $M(L)$ accepted by a pushdown automaton

$M = (\sum, Q, s, I, \gamma, F)$, is a context-free

language, where \sum is a finite alphabet, Q is a finite set of states, s is the initial state, I is a finite of stack symbols, γ is the transition relation and F is the set of acceptance states.