Arts
CBCS/2025
HE/6th Sem

3 (Sem-6/CBCS) MAT HE 4

2025

MATHEMATICS

(Honours Elective)

Paper : MAT–HE–6046

(Hydromechanics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 10 = 10$

   (i) Define specific heat of a body.

   (ii) Define surface of equal pressure.

   (iii) For an irrotational flow $curl\,\bar{q} \neq 0$. *(State True **or** False)*

   (iv) State Charle's law.

   (v) Name the instrument used to measure atmospheric pressure.

   (vi) What is tube of flow ?

(vii) What is the physical meaning of $curl \vec{q}$?

(viii) Define absolute zero of temperature.

(ix) Name the *two* methods of treating the general problem of Hydrodynamics.

(x) What is meant by rotational flow ?

2. Answer the following questions : 2×5=10

(a) Show that the surface of equal pressure is intersected orthogonally by the lines of force.

(b) If ρ_0 and ρ be the densities of a gas at 0^0 and t^0 centigrade respectively, then establish the relation $\rho_0 = \rho(1+\alpha t)$ where $\alpha = \frac{1}{273}$.

(c) Establish the relation between local and individual time rate of change.

(d) What is meant by velocity potential? Does velocity potential exist for a rotational flow ?

(e) Define streamline and path line of a fluid particle.

3. Answer **any four** of the following questions : 5×4=20

(a) A mass of fluid is at rest under the forces $X = (y+z)^2 - x^2$, $Y = (z+x)^2 - y^2$, $Z = (x+y)^2 - z^2$; find the density and the surfaces of equal pressure.

(b) A fluid at rest is in equilibrium in the forces field $X = y^2 + z^2 - xy - zx$, $Y = z^2 + x^2 - yz - yx$, $Z = x^2 + y^2 - zx\_zy$. Show that the curves of equal pressure and density are a set of circles.

(c) If the absolute temperature T at a height z is a function of the pressures at two heights z_1 and z_2 then show that

$$log\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{f(z)}$$

R being the constant in the equation $p = R\rho T$.

(d) The velocity components of a flow in cylindrical polar coordinates are $\left(r^2 z\cos\theta, rz\sin\theta, z^2 t\right)$. Determine the components of the acceleration of a fluid particle.

(e) Test whether the motion specified by

$$\vec{q} = \frac{k^2(xi - yj)}{x^2 + y^2},\ k \text{ is constant, is a}$$

possible motion for an incompressible fluid. If so, determine the equations of streamlines.

(f) The particles of a fluid motion move symmetrically in space with regard to a fixed centre; prove that the equation of continuity is $\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial}{\partial r}\left(ur^2\right) = 0$, where u is the velocity at a distance r.

4. Answer ***any four*** parts : 10×4=40

(a) (i) Show that $\frac{x^2}{a^2}tan^2 t + \frac{y^2}{b^2}cot^2 t = 1$ is a possible form of bounding surface of a liquid.

(ii) Two volumes V_1 and V_2 of different gases at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 are mixed together, so that the volume of the mixture is V and absolute temperature is T. Prove that the pressure of the mixture is

$$\frac{T}{V}\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right).$$

(b) (i) A mass of homogeneous liquid, contained in a vessel, revolves uniformly about a vertical axis. Determine the pressure at any point and the surfaces of equal pressure.

(ii) Show that the centre of pressure of a circular area immersed in the liquid whose centre is at a depth h below the surface, when the density of the liquid varies as the depth, is at a depth $\frac{2a^2h}{a^2 + 4h^2}$ below the centre of the circle, where a is the radius of the circular area.

(c) (i) A liquid of given volume V is at rest under the forces

$X = \frac{\mu x}{a^2}, Y = -\frac{\mu y}{b^2}, Z = -\frac{\mu z}{c^2}$. Find the pressure at any point of the liquid and the surfaces of equal pressure.

(ii) For a perfect gas, establish the relation $C_p - C_v = R$, where the symbols have their usual meanings.

(d) Obtain Euler's equation of motion for a non-viscous fluid in the form

$$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}P.$$

(e) Obtain the equation of continuity for a fluid in motion in the form

$\frac{\partial p}{\partial t} + div(P\vec{q}) = 0$ where p and $\vec{q}$ are repectively the density and the velocity of the fluid. Deduce the form of equation of continuity when the fluid is homogeneous and incompressible.

(f) Stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are D and d; if V and v be the corresponding velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone,

prove that $\frac{v}{V} = \frac{D^2}{d^2} e^{\frac{v^2 - V^2}{2k}}$ where k is the pressure divided by the density and suppose constant.

(g) Prove that if the forces per unit of mass at $(x, y; z)$ parallel to the axes are $y(a-z), x(a-z), xy$; the surface of equal pressure are hyperbolic paraboloid and the curves of equal pressure and density are rectangular hyperbolas.

(h) A hemispherical bowl is filled with water and two vertical planes are drawn through its central radius, cutting off a semilune of the surface, If 2α be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical is

$$tan^{-1}\left(\frac{\sin\alpha}{\alpha}\right)$$