

Total number of printed pages-7

3 (Sem-6/CBCS) MAT HC 2

2024

MATHEMATICS

(Honours Core)

Paper : MAT-HC-6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following: $1 \times 7 = 7$

(i) Under which of the following conditions does arbitrary constant elimination usually produce more than one partial differential equation of order one ?

(a) The number of arbitrary constants is less than that of independent variables

Contd.

- (b) The number of arbitrary constants equals the number of independent variables
- (c) The number of arbitrary constants is more than that of independent variables
- (d) Both (a) and (b)

(Choose the correct answer)

(ii) State True or False :

$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ is a first order quasi-linear partial differential equation.

(iii) The order of $p \tan y + q \tan x = \sec^2 z$ is _____.

(iv) The Charpit's method is used for

- (a) general solution
 (b) complete solution
 (c) singular solution
 (d) complete integral

(Choose the correct answer)

(v) Jacobi's auxiliary equations for $p_1x_1 + p_2x_2 - p_3^2 = 0$ are _____.

(vi) What are the characteristic equations of $u_x - u_y = u$?

(vii) The equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$ is

- (a) parabolic for $x \neq 0$ and $y \neq 0$ only
 (b) parabolic for $x = 0$ and $y = 0$ only
 (c) parabolic everywhere
 (d) parabolic nowhere

(Choose the correct answer)

2. Answer in short :

2×4=8

(i) Consider an equation of the form $a(x, y, u)u_x + b(x, y, u)u_y = c(x, y, u)$, where its coefficients a , b and c are functions of x , y and u . Is it linear ? Justify your answer.

- (ii) Eliminate the arbitrary function f from $z = x^n f\left(\frac{y}{x}\right)$ to form a partial differential equation.
- (iii) Mention when Jacobi's method is used. Name an advantage of Jacobi's method over Charpit's method.
- (iv) Construct an example of a partial differential equation that is elliptic in one domain but hyperbolic in another.

3. Answer **any three** : 5×3=15

- (i) Find the partial differential equation that all surfaces of revolution satisfy with the z -axis as the axis of symmetry, along with a suitable explanation.
- (ii) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z.$$

- (iii) Find the integral surface of the equation $(x-y)y^2 p + (y-x)x^2 q = (x^2 + y^2)z$ through the curve $xz = a^2$, $y = 0$.
- (iv) Reduce the equation $u_x + 2xyu_y = x$ to canonical form, and obtain the general solution.
- (v) Discuss the general solution of $Au_{xx} + Bu_{xy} + Cu_{yy} = 0$ with constant coefficients in hyperbolic case.

4. Answer the following : 10×3=30

- (i) Discuss briefly the essential steps in Charpit's method for solving partial differential equations. Use this method to solve the equation $p = (z + qy)^2$.

Or

Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

- (ii) Describe in brief the key components of the 'method of separation of variables'. Use this method suitably to solve the equation $u_x + u = u_y$, $u(x, 0) = 4e^{-3x}$.

Or

Use $v = \ln u$ and $v(x, y) = f(x) + g(y)$ to solve the equation $x^2u_x^2 + y^2u_y^2 = u^2$.

Also, discuss briefly the approach adopted to solve the above equation.

- (iii) Consider the wave equation

$$u_t - c^2u_{xx} = 0, \text{ } c \text{ is constant.}$$

Establish that any general solution of this equation can be expressed as the sum of two waves, one travelling to the right with constant velocity c and the other travelling to the left with the same velocity c .

Or

Find the general solution of the following equations :

$$(a) \quad yu_{xx} + 3yu_{xy} + 3u_x = 0, \quad y \neq 0$$

$$(b) \quad 4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$$