(b) (i) A plane passes through a fixed point $(p, q, r)$ and cuts the axes at the points $A$, $B$ and $C$. Show that the locus of the centre of the sphere $OABC$ is $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=2$.

(ii) Find the equation of the cylinder whose generators are parallel to the line $2x = y = 3z$ and which passes through the circle $y = 0$, $z^2 + x^2 = 8$. 5+5=10

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**3 (Sem–3/CBCS) MAT HC 3**

**2024**

# MATHEMATICS

(Honours Core)

Paper : MAT–HC–3036

***(Analytical Geometry)***

*Full Marks : 80*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer **all** the questions : 1×10=10

(a) Find the equation to the locus of the point $P(t, 2t)$, where $t$ is a parameter.

(b) Find the eccentricity of the hyperbola $x^2 - y^2 = 1$.

(c) Find the angle between the pair of lines $x^2 - y^2 = 0$.

(d) What is the polar equation of a circle with the pole as the centre ?

(e) Under what condition does the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represent a pair of parallel straight lines ?

(f) Write down the equation of the $z$-axis in symmetric form.

(g) What are the direction cosines of the normal to the plane $2x - y + 2z = 3$ ?

(h) Find the equation of the cone whose vertex is the origin and the guiding curve is $x = a$, $y^2 + z^2 = b^2$.

(i) Define the shortest distance between two skew lines.

(j) For what value of $a$, the transformation $x' = -x + 2$, $y' = ay + 3$ is a translation ?

2. Answer **all** the questions : 2×5=10

(a) Find the value of $k$, if the equation $kxy - 8x + 9y - 12 = 0$ represents a pair of straight lines.



(b) If the axes are rotated through an angle $\tan^{-1}2$, what does the equation $4xy - 3x^2 = a^2$ become ?

(c) The axes of a right circular cylinder is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$ and the radius is

5. Find the equation of the cylinder.

(d) If $e_1$ and $e_2$ are the eccentricities of a hyperbola and its conjugate, show that

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1.$$

(e) Find the equation of the sphere passing through the circles $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

3. Answer **any four** questions : 5×4=20

(a) If by rotation of axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.

(b) Deduce the polar equation of a conic with the focus as the pole.

(c) Find the equation of the tangent to the hyperbola $4x^2 - 9y^2 = 1$ which is parallel to the line $4y = 5x + 7$.

(d) Prove that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

(e) Show that the equation of the cone whose vertex is the origin and the guiding curve is $z = k$, $f(x, y) = 0$, is

$$f\left(\frac{kx}{z}, \frac{ky}{z}\right) = 0.$$

(f) Find the equation of the director sphere of the conicoid $ax^2 + by^2 + cz^2 = 1$.

*Answer* ***either*** *(a)* ***or*** *(b) from the following questions:* 10×4=40

4. (a) (i) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines equidistant from the origin, then show that

$$f^4 - g^4 = c\left(bf^2 - ag^2\right).$$



(ii) Find the lengths of semi-axes of the conic $ax^2 + 2hxy + by^2 = 1$. 5+5=10

(b) (i) Find the asymptotes of the hyperbola $xy + ax + by = 0$.

(ii) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to the standard form. 5+5=10

5. (a) (i) Show that the line $lx + my = n$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $a^2l^2 + b^2m^2 = n^2$.

(ii) Show that the locus of the points of intersection of perpendicular is its directrix. 5+5=10

(b) (i) If the chord $PP'$ of a hyperbola meets the asymptotes at $Q$ and $Q'$, then show that $PQ = P'Q'$.

(ii) If $PSP'$ and $QSQ'$ are two perpendicular focal chords of a conic, prove that

$$\frac{1}{PS.SP'}+\frac{1}{QS.SQ'}=a \text{ (constant)}.$$

5+5=10

6. (a) (i) Deduce the expression of the shortest distance between the skew lines

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n},$$

$$\frac{x-\alpha'}{l'}=\frac{y-\beta'}{m'}=\frac{z-\gamma'}{n'}.$$

(ii) A variable plane is parallel to the given plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes at the points $A$, $B$, $C$ respectively. Prove that the circle $ABC$ lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$$

5+5=10



(b) (i) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

(ii) Prove that the locus of the poles of a tangent plane to the conicoid $ax^2+by^2+cz^2=1$ with respect to the conicoid $\alpha x^2+\beta y^2+\gamma z^2=1$ is the conicoid $\frac{\alpha^2x^2}{a}+\frac{\beta^2y^2}{b}+\frac{\gamma^2z^2}{c}=1$.

5+5=10

7. (a) (i) Show that the director sphere of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is the sphere $x^2+y^2+z^2=a^2+b^2+c^2$.

(ii) Obtain the equation of the chord of the conic $\frac{l}{r}=1+e\cos\theta$, joining the two points on the conic, whose vectorial angles are $(\alpha+\beta)$ and $(\alpha-\beta)$.

5+5=10