(h) (i) Consider the following assignment problem where 5 different jobs are to be assigned to 5 different operators such that the total processing time is minimized . The matrix entries represent processing times in hours.

Job \ Operator	1	2	3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

Develop a zero-one programming model. 5

(ii) What is an unbalanced assignment problem ? Explain the difference between transportation problem and assignment problem. 2+3=5

Total number of printed pages–16

3 (Sem–3/CBCS) MAT HG 1/2/RC

2024

MATHEMATICS

(Honours Generic/Regular)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HG–3016 /MAT–RC–3016

(Differential Equation)

OPTION–B

Paper : MAT–HG–3026

(Linear Programming)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION–A

Paper : MAT–HG–3016 /MAT–RC–3016

(Differential Equation)

*Answer **either** in English **or** in Assamese.*

1. Answer the following questions : 1×10=10
তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ

(a) Write the condition of exactness of an ordinary differential equation.
এটা সাধাৰণ অৱকল সমীকৰণ যথার্থ হোৱাৰ চর্ত লিখা।

(b) Find Wronskian of $y_1 = e^{2t}$, $y_2 = e^{-3t}$.
$y_1 = e^{2t}$ আৰু $y_2 = e^{-3t}$ ৰ Wronskian নির্ণয় কৰা।

(c) Find integrating factor :
অনুকলন গুণক নির্ণয় কৰা ঃ

$$\frac{dy}{dx} - \frac{y}{x^2} = \frac{1}{x}$$

(d) Write complementary function :
সমীকৰণটোৰ পৰিপূৰক ফলনটো লিখা ঃ

$$\left(D^2 + 5\right)y = e^{2x}$$

(e) Write standard equation of first order linear differential equation.
প্ৰথম ক্ৰমৰ ৰৈখিক অৱকল সমীকৰণৰ মানক আর্হি লিখা।

(f) Write particular integral of :
নির্দিষ্ট অনুকলন লিখা ঃ

$$D^3y = e^x$$

(g) Obtain differential equation of :
অৱকল সমীকৰণ নির্ণয় কৰা ঃ

$$by = cx$$

(h) What do you mean by orthogonal trajectories of a family of curves ?
এটা বক্ৰ সমষ্টিৰ লাম্বিক প্ৰক্ষেপপথ বুলিলে কি বুজা ?

(i) Write solution of following simultaneous differential equations :
তলৰ যোৰ অৱকল সমীকৰণবোৰৰ সমাধান লিখা ঃ

$$dy/dt = 0, \quad dx/dt = 2$$

(j) Solve :
সমাধান কৰা ঃ

$$d^2y/dx^2 = 0$$

2. Answer the following questions : 2×5=10

তলৰ প্ৰশ্নবোৰ সমাধা কৰা ঃ

(a) Find particular integral of :

নিৰ্দিষ্ট অনুকলন লিখা ঃ

$$(D^4+1)y = \sin x$$

(b) Solve :

সমাধা কৰা ঃ

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

(c) Solve :

সমাধা কৰা ঃ

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

(d) Solve :

সমাধা কৰা ঃ

$$(x^2+y^2)dx + xy\,dy = 0$$

(e) Find orthogonal trajectories :

লাম্বিক প্ৰক্ষেপপথ উলিওৱা ঃ

$$y = cx$$

3. Answer the following : ***(any four)*** 5×4=20

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰা ঃ *(যিকোনো চাৰিটা)*

(a) Find the orthogonal trajectory of the family of circles passing through the origin and centre on x-axis.

মূলবিন্দুৰ মাজেৰে যোৱা আৰু x-অক্ষৰ ওপৰত কেন্দ্ৰ থকা বৃত্তবৃন্দৰ লাম্বিক প্ৰক্ষেপপথ নিৰ্ণয় কৰা।

(b) Solve :

সমাধা কৰা ঃ

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3$$

(c) Solve :

সমাধা কৰা ঃ

$$y(1+xy)dx + x(1-xy)dy = 0$$

(d) Solve :

সমাধা কৰা ঃ

$$\frac{dy}{dx} + \frac{1}{x} = e^y/x^2$$

(e) Solve :

সমাধা কৰা ঃ

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$

(f) Solve :
সমাধা কৰা ঃ

$$y = yp^2 + 2px$$

4. Answer ***any four*** : 10×4=40
যিকোনো চাৰিটা সমাধা কৰা ঃ

(a) Solve by the method of variation of parameter :
প্ৰাচল বিচৰণ পদ্ধতিৰে সমাধা কৰা ঃ

$$\frac{d^2y}{dx^2} + y = \tan x$$

(b) Describe the method of solving the exact equation $Mdx + Ndy = 0$.
যথাৰ্থ অৱকল সমীকৰণ $Mdx + Ndy = 0$-ৰ সমাধা পদ্ধতি বৰ্ণনা কৰা।

(c) Solve :
সমাধা কৰা ঃ

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 4\log x$$

(d) Solve :
সমাধা কৰা ঃ

$$\frac{dx}{dt} + y = e^t; \quad x - \frac{dy}{dt} = t$$

(e) Solve by reducing to normal form :

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$$

নৰ্মাল ৰূপলৈ সমানীত কৰি সমাধা কৰা ঃ

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$$

(f) Solve :
সমাধা কৰা ঃ

$$(D^2 + 4D + 4)y = x\sin 2x$$

(g) Discuss the method of finding auxiliary equation of an ordinary differential equation. What is the importance of particular integral in the solution of differential equation ?
এটা সাধাৰণ অৱকল সমীকৰণৰ 'অক্‌জিলাৰী' সমীকৰণ উলিওৱা পদ্ধতিটো বৰ্ণনা কৰা। অৱকল সমীকৰণটোৰ সমাধানত এটা নিৰ্দিষ্ট অনুকলনৰ প্ৰয়োজনীয়তা কি ?

(h) Discuss the method of variation of parameters.
"Variation of parameter" পদ্ধতিটো বৰ্ণনা কৰা।

OPTION-B

Paper : MAT-HG-3026

(Linear Programming)

1. Answer the following questions : *(Choose the correct answer)* 1×10=10

(a) A point $\bar{x} \in X$ is an extreme point of a convex set X iff there do not exist points $\bar{x}_1, \bar{x}_2 (\bar{x}_1 \neq \bar{x}_2)$ in X such that

(i) $\bar{x} = (1-\lambda)\bar{x}_1 + \lambda\bar{x}_2, \quad 0 < \lambda < 1$

(ii) $\bar{x} = (1-\lambda)\bar{x}_1 + \lambda\bar{x}_2, \quad 0 \leq \lambda \leq 1$

(iii) $\bar{x} = (1-\lambda)\bar{x}_1 + \lambda\bar{x}_2, \quad \lambda$ real

(iv) None of the above

(b) A set of feasible solutions in a linear programming problem is

(i) non-convex set

(ii) convex set

(iii) disconnected set

(iv) concave set

(c) In a given LPP, we have the objective function

$$Z = \sum_{j=1}^{r} c_j x_j + C, \text{ then}$$

(i) the above statement is always true

(ii) the above statement is always false

(iii) partially true

(iv) None of the above

(d) What is the primary purpose of the simplex method ?

(i) To find an initial feasible solution

(ii) To find an optimal feasible solution

(iii) To determine the number of iterations

(iv) None of the above

(e) Let us consider the system of linear equations

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

Then $\left(0, \frac{5}{3}, -\frac{1}{3}\right)$ is

(i) a feasible solution

(ii) a basic feasible solution

(iii) a basic solution

(iv) not a solution

(f) An LPP is defined as

Minimize $Z = 15x_1 + 12x_2$

subject to

$$x_1 + 2x_2 \leq 3$$
$$2x_1 - 4x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

The objective function of the dual of this LPP is

(i) Maximize $W = y_1 + y_2$

(ii) Maximize $W = y_1 + 2y_2$

(iii) Maximize $W = 2y_1 - 4y_2$

(iv) Maximize $W = 3y_1 + 5y_2$

(g) What is the primary objective of the transportation problem ?

(i) Minimize transportation cost

(ii) Maximize profit

(iii) Optimize inventory levels

(iv) Balance supply and demand

(h) Which method is commonly used to solve assignment problem ?

(i) Simplex method

(ii) Hungarian method

(iii) Dual simplex method

(iv) Branch and bound method

(i) In two-person zero-sum game

(i) gain by one player is equal to loss by the other player

(ii) two players gain

(iii) gain by one player is more than the loss by other player

(iv) None of the above

(j) If $A = [a_{ij}]$ is the pay off matrix, then saddle point exists when

(i) $\underset{j}{Min}\ \underset{i}{max}\ a_{ij} \leq \underset{i}{max}\ \underset{j}{mix}\ a_{ij}$

(ii) $\underset{j}{Min}\ \underset{i}{max}\ a_{ij} \geq \underset{i}{max}\ \underset{j}{mix}\ a_{ij}$

(iii) $\underset{j}{Min}\ \underset{i}{max}\ a_{ij} = \underset{i}{max}\ \underset{j}{mix}\ a_{ij}$

(iv) None of the above

2. Answer the following : 2×5=10

(a) Show that a hyperplane is a convex set.

(b) How do the graphical and simplex methods of solving LPP differ from each other ?

(c) Form the dual of the following primal problem :

Minimize $Z = 20x_1 + 40x_2$

subject to

$$2x_1 + 20x_2 \geq 40$$
$$20x_1 + 3x_2 \geq 20$$
$$4x_1 + 15x_2 \geq 30$$
$$x_1 \text{ and } x_2 \geq 0$$

(d) State the mathematical formulation of an assignment problem.

(e) Explain the difference between pure and mixed strategy in the context of game theory.

3. Answer ***any four*** of the following : 5×4=20

(a) Define convex set and show that the intersection of any finite number of convex sets is a convex set.

(b) Solve the following LPP using graphical method :

Maximize $Z = 6x_1 + 8x_2$

subject to

$5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

x_1 and $x_2 \geq 0$

(c) Solve the following LPP using Big M method :

Minimize $Z = 10x_1 + 15x_2 + 20x_3$

subject to

$2x_1 + 4x_2 + 6x_3 \geq 24$

$3x_1 + 9x_2 + 6x_3 \geq 30$

$x_1, x_2, x_3 \geq 0$

(d) What is duality? What is the significance of dual variables in a linear programming model? 2+3=5

(e) Solve the following assignment problem using Hungarian method (The matrix entries are processing times in hours) :

Job \ Operator	1	2	3	4	5
1	20	22	35	22	18
2	4	26	24	24	7
3	23	14	17	19	19
4	17	15	16	18	15
5	16	19	21	19	25

(f) The pay-off matrix of a two-person zero-sum game is given below :

A \ B	*I*	*II*	*III*	*IV*	*V*
I	9	3	1	8	0
II	6	5	4	6	7
III	2	4	3	3	8
IV	5	6	2	2	1

Find the best strategy for each player and the value of the game.

4. Answer ***any four*** questions : 10×4=40

(a) Show that the following system of linear equations has degenerate solution :

$2x_1 + x_2 - x_3 = 2$

$3x_1 + 2x_2 + x_3 = 3$

(b) Anita Electric Company produces two products P_1 and P_2. Products are produced and sold on a weekly basis. The weekly production cannot exceed 25 for product P_1 and 35 for product P_2 because of limited available facilities. The company employs total of 60 workers. Product P_1 requires 2 man – weeks of labour, while P_2 requires one man – week of labour. Profit margin on P_1 is Rs 60 and on P_2 is Rs 40. Formulate this problem as an LPP and solve that using graphical method.

(c) Solve the following LPP using Simplex method :

Maximize $Z = 5x_1 + 3x_2 + 7x_3$

subject to

$$x_1 + x_2 + 2x_3 \le 22$$
$$3x_1 + 2x_2 + x_3 \le 26$$
$$x_1 + x_2 + x_3 \le 18$$
$$x_1,\ x_2 \text{ and } x_3 \ge 0$$

(d) Solve the following LPP using two-phase method :

Minimize $Z = 12x_1 + 18x_2 + 15x_3$

subject to

$$4x_1 + 8x_2 + 6x_3 \ge 64$$
$$3x_1 + 6x_2 + 12x_3 \ge 96$$
$$x_1,\ x_2 \text{ and } x_3 \ge 0$$

(e) Find the initial basic feasible solution to the following transportation problem by

(i) Northwest corner cell method and

(ii) Least cost cell method

From \ To	1	2	3	Supply
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	2	9	18	

State which of the methods is better.

(f) Consider the pay-off matrix of player A and solve it optimally using the graphical method :

Player A \ Player B	1	2	3	4	5
1	3	6	8	4	4
2	–7	4	2	10	2

(g) (i) What is the objective of the transportation problem? 2

(ii) Show that a transportation. problem is a special type of LPP. 4

(iii) With reference to a transportation problem define the following terms : Basic feasible solution, Optimal solution. 2+2=4